

Online Evaluation of Measurement Uncertainty in Sensor Networks: A Case Study on Voltage Transformers

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Abstract—Sensor networks serve as the perceptual core of industrial systems, such as the Internet of Things and smart grid, where the accuracy of individual measurements is pivotal to ensuring reliable state estimation. Nevertheless, factors, such as environmental interference and device aging, introduce measurement uncertainties to sensors, and the incompleteness of the information would propagate and accumulate across the network. Despite advances in evaluating the measurement uncertainty of individual sensors and incomplete assessments of the measurement status in sensor networks, network-level evaluation of measurement uncertainty remains an open yet practical problem. Thus, we propose a recursive framework for online evaluation of measurement uncertainties within sensor networks, demonstrated through a case study on voltage transformers in the smart grid. The proposed method incorporates a measurement model to capture interdevice dependencies and utilizes the Monte Carlo method to propagate parameter distributions considering the measurement uncertainties. Next, a recursive workflow for evaluating measurement uncertainties across the entire sensor network is proposed. Specifically, the calibrated nodes are selected based on the uncertainty propagation theory, and then the Bayesian fusion is applied to estimate the parameter distributions of extended nodes. Simulations conducted on the IEEE 30-node system, supplemented by real-world validation with power system data, indicate that the proposed method is capable of achieving high evaluation accuracy and stable uncertainty propagation.

Index Terms—Measurement uncertainty, online evaluation, sensor network, voltage transformer (VT).

NOMENCLATURE

Z	Series impedance of the transmission line.
Y	Shunt admittance of the transmission line.

V_s	Upstream voltage of the transmission line.
I_s	Upstream current of the transmission line.
V_r	Downstream voltage of the transmission line.
I_r	Downstream current of the transmission line.
V_{sm}	Voltage measured by upstream voltage transformer.
I_{sm}	Current measured by upstream current transformer.
V_{rm}	Voltage measured by downstream voltage transformer.
I_{rm}	Current measured by downstream current transformer.
K_{vs}	Correction factor of upstream voltage transformer.
K_{is}	Correction factor of upstream current transformer.
K_{vr}	Correction factor of downstream voltage transformer.
K_{ir}	Correction factor of downstream current transformer.
f_{vs}	Ratio error of upstream voltage transformer.
δ_{vs}	Phase displacement of upstream voltage transformer.

The superscripts of variables can be A/B/C to represent phase sequences, or P/N/Z to represent positive, negative, or zero components.

I. INTRODUCTION

SENSOR networks, composed of numerous distributed nodes, perform real-time monitoring and data acquisition of critical variables in complex systems [1]. They constitute the perceptual layer of industrial systems, including the Internet of Things (IoT), smart grid, etc. [2]. Without loss of generality, each sensor in a node of the sensor network converts inaccessible physical quantities into measurable signals via the specialized transduction model. During sensor deployment, factors, such as device aging and environmental interference, can cause parameter drift in the transduction model, which in turn leads to measurement drift [3]. Such sensing performance degradation is denoted as the *measurement uncertainty*, providing a quantitative metric of completeness [4]. Such adverse effects will propagate through the data flows of the sensor network, potentially leading to cumulative distortions that degrade the accuracy of various key applications, such as state estimation, decision-making, and intelligent analytics [5]. Specifically, recent IoT trust management frameworks, such as StackTrust [6], focus on secure communication in large-scale sensing systems, highlighting that measurement uncertainty is relevant and complementary. Thus, modeling and evaluating measurement uncertainties in sensor networks are crucial for ensuring system reliability [7].

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Existing methods for evaluating the measurement status of sensors can be roughly categorized into single-device and group-based methods [8]. Single-device methods focus on characterizing the measurement uncertainty of an individual sensor through laboratory calibration, modeling of transduction processes, or statistical analysis of repeated measurements [9], [10]. Notably, the calibration methods are constrained by experimental conditions and, thus, are not applicable for online evaluation; moreover, all these methods cannot capture the influence of interdevice interactions to achieve network-level evaluation. In contrast, group-based methods exploit correlations or redundancy among a group of sensors within a sensor network, leveraging data analysis techniques, e.g., principal component analysis, to assess measurement status at the network level. Nevertheless, their current application remains largely qualitative for fault detection rather than measurement uncertainty evaluation [11], [12].

In the field of the smart grid, voltage transformers (VTs) deployed across different substations form a critical and representative sensor network. On the one hand, VTs serve as core sensing units for high voltage measurement, assessing their network-level measurement status is essential for maintaining power system stability and ensuring equitable energy transactions [13]. On the other hand, the sensor network formed by VTs exhibits wide spatial distribution, strong interconnectivity, and strict accuracy requirements, thereby embodying the characteristic features of a typical sensor network. Consequently, realizing the measurement status evaluation of VT-based sensor networks provides valuable insights and reference for reliability maintenance of other sensor networks. Many efforts have been made to measurement status evaluation of VTs from the perspective of sensor networks. These methods typically involve constructing equations based on transmission line models, utilizing calibrated VTs to estimate the parameters of uncalibrated units. For instance, early research focused on using independent transmission line models under different assumptions [14], [15]. Later, local transmission line models with VT interaction were developed by incorporating bus voltage consistency and Kirchhoff's Current Law to extend evaluation to local grids [16]. Meanwhile, a unified regional framework was proposed to characterize the interactions among all the VTs in the same network for the simultaneous evaluation [17]. The above approaches target only the measurement bias of VTs, which represent merely a point estimate of the measurement uncertainty arising from stochastic factors and calibration devices, resulting in information incompleteness.

Existing methods for quantitatively assessing the condition of sensor networks focus on evaluating and calibrating measurement bias, whereas measurement uncertainty has received limited attention. However, measurement uncertainty is essential both for quantifying the quality of measurement results [4] and for characterizing the metrological condition of sensing devices [18], motivating its explicit quantification in sensor network condition assessment. Thus, we propose an online evaluation method and demonstrate it through a case study on VTs in the smart grid. Notably, the term online is used in contrast to offline calibration, indicating the noninterruptive and real-time

nature of the method. The distribution of the transduction model parameter is defined as a composite of both the estimated parameter and uncertainty, with innovative improvements introduced in model formulation and propagation mechanisms. The key contributions of this work are summarized as follows.

- 1) A measurement model (MM) is developed to characterize the relationships between transduction model parameters of adjacent sensors. Then, parameter distribution propagation is facilitated to support network-level evaluation of measurement uncertainties.
- 2) A recursive workflow is proposed for online evaluation of measurement uncertainties in sensor networks. It incorporates uncertainty propagation theory to identify calibrated nodes with the minimum uncertainty increment and subsequently estimates the parameter distributions of extended nodes, thereby achieving reliable evaluations.
- 3) Simulations on the IEEE 30-node model indicate the high evaluation accuracy of the proposed method with tolerable uncertainties, as well as effectively propagating measurement uncertainties over extended distances. Real-world validation demonstrates high consistency between the evaluated and calibrated results for both parameters and uncertainties, indicating the practical applicability of the proposed method.

The rest of this article is organized as follows. Section II outlines the problem context and theoretical background. Section III presents the proposed method. Section IV validates the effectiveness of the proposed method via simulation and real-world experiments. Section V discusses the method's validation behavior and its feasibility, adaptability, and deployment potential in general sensor networks. Finally, Section VI concludes this article.

II. BACKGROUND

The background of the transduction model and measurement uncertainty of the VT is first illustrated. Then, the principle of Bayesian fusion (BF) for distribution estimation is also presented.

A. Transduction Model

A transduction model characterizes the input–output relationship of a sensor. Here, we take VTs as an example to illustrate the concept of the transduction model. Three-phase VTs are deployed within substations interconnected by transmission lines, as illustrated in Fig. 1. The transduction model of a VT can be characterized by its transformation ratio k_r , indicating the ratio between the secondary (output) and primary (input) voltages, i.e.,

$$k_r = \frac{V_2}{V_1}. \quad (1)$$

Here, V denotes voltage,¹ and subscripts 1 and 2 indicate the primary and secondary sides, respectively. The transformation ratio can be further expressed relative to its rated value by

¹Unless specified, all electrical quantities in this article are phasors, and j in $e^{j\delta}$ denotes the imaginary unit.

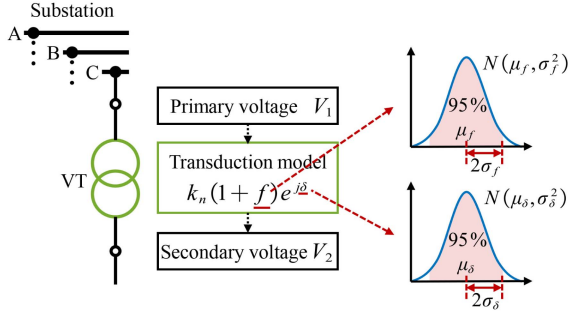


Fig. 1. Diagram of the voltage measurement by a VT in the smart grid, where the measurement uncertainty characterized by a distribution is rooted in the transduction model.

incorporating both ratio error (RE) and phase displacement (PD) as the transduction model parameters. RE quantifies the extent to which the transformation ratio deviates from the rated value, typically expressed as a percentage. PD reflects the phase angle difference between primary and secondary quantities, usually measured in degrees or minutes of arc. Let f and δ denote the RE and PD, we have

$$k_r = k_n(1 + f)e^{j\delta} \quad (2)$$

with k_n denoting the rated transformation ratio. In practice, when the secondary voltage is used to derive the primary voltage to be measured, the result can be expressed as

$$V_{1m} = \frac{V_2}{k_n} \quad (3)$$

where subscript m denotes the measured value corresponding to the true quantity. By combining (1)–(3), we can derive

$$V_1 = \frac{1}{(1 + f)e^{j\delta}} V_{1m}. \quad (4)$$

Thus, f and δ can characterize the deviation between the measured and true values. Subsequently, we can define

$$K = \frac{V_1}{V_{1m}} = \frac{1}{(1 + f)e^{j\delta}} \quad (5)$$

as the correction factor considering both f and δ [19]. The above derivation is applicable to current transformers (CTs) used for measuring terminal currents of transmission lines. Notably, phasor measurement units (PMUs) are typically connected downstream of VTs/CTs to convert their outputs into synchronized phasors.

B. Measurement Uncertainty

Measurement uncertainty is a parameter associated with a measurement result that quantifies the degree of dispersion between the measured value and the true value [4]. In the context of this study, we focus on the inherent measurement condition of the sensor, which is determined by the uncertainties of the transduction model parameters [20]. Assuming a specific parameter of the transduction model follows a Gaussian normal distribution $N(\mu_0, \sigma_0^2)$, the half-width of the coverage interval represents the uncertainty. Specifically, the half-width of the 95% coverage interval corresponds to approximately $2\sigma_0$,

as illustrated in Fig. 1. The reason for choosing Gaussian distributions is that, according to the guide to the expression of uncertainty in measurement (GUM) [20], assuming uniform or Gaussian distributions is acceptable when prior knowledge is limited, and VT studies report that transduction parameters typically exhibit Gaussian characteristics [21]. Moreover, our framework is compatible with arbitrary parameter-distribution shapes. These distributions then serve as the medium for propagating uncertainty within the proposed evaluation method, enabling a network-level measurement uncertainty evaluation.

To address uncertainty in practical applications, two primary approaches have been developed. The GUM, issued by the Joint Committee for Guides in Metrology, provides a standardized framework for the evaluation and reporting of measurement uncertainty [20]. In GUM, uncertainties associated with individual input quantities are propagated and combined using the uncertainty propagation theory, yielding the overall uncertainty of the output quantity [22]. However, the validity of this method is contingent upon the MM being inherently linear or appropriately linearized using a first-order Taylor expansion [23].

The Monte Carlo method (MCM) is recommended in GUM Supplement 1 [24], which is tailored for MMs characterized by significant nonlinearity. The MCM is a numerical simulation technique based on stochastic sampling, in which random samples from the probability density functions (PDFs) of input quantities are propagated through the MM to generate outputs. This approach then constructs the resulting output distribution, providing an evaluation of measurement uncertainty [25]. While MCM exhibits strong adaptability in handling complex distributions and nonlinear models, it is often criticized for its limited interpretability. In certain cases, linearized models are still employed as auxiliary tools to validate the results generated by MCM [26].

In addition to the aforementioned GUM and MCM, there are other approaches for evaluating measurement uncertainty, such as fuzzy and machine-learning-based methods. Specifically, fuzzy methods represent uncertainty using membership functions to handle imprecise or vague information, while machine-learning-based methods leverage data-driven models and propagate measurement uncertainty through learned relationships [27], [28]. Notably, according to existing studies and metrology guidelines, the GUM and MCM remain the most accurate and accepted techniques for the vast majority of MMs [29].

C. Bayesian Fusion

The BF estimates the distribution of unknown variable θ by integrating observations from n information sources $\mathbf{x} = (x_1, x_2, \dots, x_n)$ as its posterior distribution [30]. If the information sources are mutually independent and satisfy $x_i \sim N(\theta, \sigma_i^2)$, the joint likelihood PDF is

$$p(\mathbf{x}|\theta) = \prod_{i=1}^n p(x_i|\theta) = k_1 \cdot \exp\left(-\sum_{i=1}^n \frac{(x_i - \theta)^2}{2\sigma_i^2}\right) \quad (6)$$

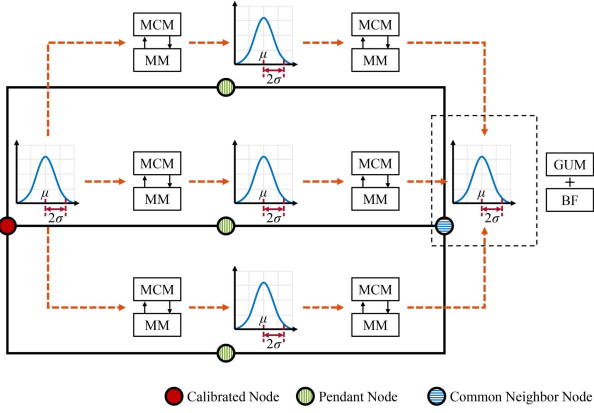


Fig. 2. Schematic of the online evaluation method. The solid lines indicate the connections between sensors, while the dashed arrows denote the propagation pathways.

with k_1 being a constant. Then, it can be derived from Bayes' theorem that

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)p(\theta)}{p(\mathbf{x})} = k_2 \cdot \exp\left(-\sum_{i=1}^n \frac{(x_i - \theta)^2}{2\sigma_i^2}\right) \quad (7)$$

with k_2 being another constant. Assuming that $\theta \sim N(\mu_0, \sigma_0^2)$, the posterior PDF of θ is

$$p(\theta|\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(-\frac{(\theta - \mu_0)^2}{2\sigma_0^2}\right). \quad (8)$$

By combining (7) and (8), we can derive the unknown variable's distribution based on multiple information sources as

$$\begin{cases} \mu_0 = \frac{\sum_{i=1}^n x_i / \sigma_i^2}{\sum_{i=1}^n 1 / \sigma_i^2} \\ \sigma_0^2 = \frac{1}{\sum_{i=1}^n 1 / \sigma_i^2} \end{cases} \quad (9)$$

When the known quantities are the mean μ_i and standard deviation σ_i provided by the information sources, x_i can be substituted for μ_i .

III. PROPOSED METHOD

The detailed method for the online evaluation of measurement uncertainties in sensor networks is elaborated, as shown in Fig. 2. First, an MM is constructed to capture the relationships between transduction model parameters of adjacent sensors. Second, the MCM is employed to propagate parameter distributions between adjacent sensors. Afterwards, a recursive workflow is designed to estimate the parameter distributions of sensors within the target region of the network. Specifically, the parameter distributions of the common neighbor nodes are obtained by hybridizing the GUM and BF methods, enabling uncertainty evaluation based on multiple calibrated nodes.

A. Measurement Model

We propose the MM to characterize the relationship between transduction model parameters of adjacent sensors, linking them as input and output, respectively. A schematic of the MM is

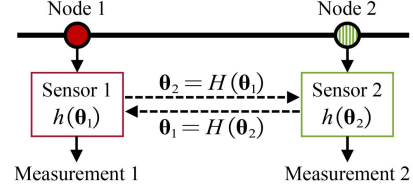


Fig. 3. Schematic of the MM H of the transduction model parameters. Here, h and θ denote the transduction model and its parameters for the sensor.

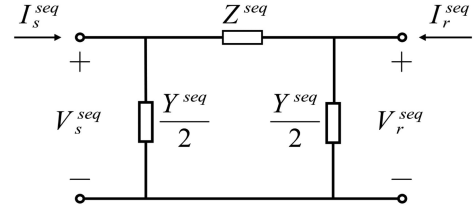


Fig. 4. Schematic of each sequence network. Here, seq denotes the positive, negative, or zero-sequence.

shown in Fig. 3, where h and θ denote the transduction model and its parameters, while H represents the MM.

The case of VTs within the substation is used as an example to show the general idea. The Nomenclature is provided to enhance readability.

Specifically, upstream and downstream substations interconnected by a transmission line are regarded as adjacent nodes, while the VTs deployed within each substation are treated as sensors. It is assumed that the downstream VTs have been calibrated, i.e., their parameters are already known. A three-phase transmission network under steady-state conditions can be equivalent to the superposition of decoupled positive, negative, and zero-sequence networks [31]. Each sequence network is characterized by the lumped parameters, as illustrated in Fig. 4. For instance, in the positive-sequence network, it can be derived based on Kirchhoff's voltage laws that

$$\begin{cases} V_r^P - V_s^P - Z^P(I_r^P - \frac{Y^P}{2}V_r^P) = 0 \\ V_s^P - V_r^P - Z^P(I_s^P - \frac{Y^P}{2}V_s^P) = 0 \end{cases} \quad (10)$$

All quantities involved in (10) are true values. Replace true values with corresponding correction factors and measured values, then (10) can be reformulated as

$$\begin{cases} K_{vr}^P V_{rm}^P - K_{vs}^P V_{sm}^P - Z^P(K_{ir}^P I_{rm}^P - \frac{Y^P}{2}K_{vr}^P V_{rm}^P) = 0 \\ K_{vs}^P V_{sm}^P - K_{vr}^P V_{rm}^P - Z^P(K_{is}^P I_{sm}^P - \frac{Y^P}{2}K_{vs}^P V_{sm}^P) = 0 \end{cases} \quad (11)$$

which can be further rewritten in matrix form as

$$\begin{bmatrix} V_{sm}^P & I_{rm}^P & 0 & 0 \\ 0 & 0 & V_{sm}^P & -I_{sm}^P \end{bmatrix} \begin{bmatrix} \frac{K_{vs}^P}{1 + \frac{1}{2}Y^P Z^P} \\ \frac{Z^P K_{ir}^P}{1 + \frac{1}{2}Y^P Z^P} \\ (1 + \frac{1}{2}Y^P Z^P) K_{vs}^P \\ Z^P K_{is}^P \end{bmatrix} = \begin{bmatrix} K_{vr}^P V_{rm}^P \\ K_{vr}^P V_{rm}^P \end{bmatrix} \quad (12)$$

Equation (12) is of the form $\mathbf{G}^P \mathbf{X}^P = \mathbf{V}^P$, where $\mathbf{G}^P$ and $\mathbf{V}^P$ are matrices composed of known quantities, and $\mathbf{X}^P$ is a matrix of unknowns. For the time-series data acquired by PMUs, data from n_t distinct time points are sampled at regular intervals to ensure that each corresponds to a different system state [15]. The dimension of $\mathbf{G}^P$ is extended to $2n_t \times 4$ and that of $\mathbf{V}^P$ is extended to $2n_t \times 1$. $\mathbf{X}^P$ can be obtained using the least-squares method, accordingly

$$\begin{cases} K_{vs}^P = \sqrt{\mathbf{X}^P(1) \cdot \mathbf{X}^P(3)} \\ Z^P Y^P = 2 \left(\sqrt{\frac{\mathbf{X}^P(3)}{\mathbf{X}^P(1)}} - 1 \right) \end{cases} \quad (13)$$

Similarly, corresponding results can be obtained based on the negative and zero-sequence networks. To decouple the three-phase electrical quantities into positive, negative, and zero-sequence components, we define

$$\mathbf{T} = \frac{1}{3} \begin{bmatrix} 1 & e^{j\frac{2}{3}\pi} & e^{j\frac{4}{3}\pi} \\ 1 & e^{j\frac{4}{3}\pi} & e^{j\frac{2}{3}\pi} \\ 1 & 1 & 1 \end{bmatrix}. \quad (14)$$

Then, we can derive that

$$\begin{bmatrix} V_s^A \\ V_s^B \\ V_s^C \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} V_s^P \\ V_s^N \\ V_s^Z \end{bmatrix}. \quad (15)$$

Replace true values with corresponding correction factors and measured values, then (15) can be reformulated as

$$\begin{bmatrix} V_{sm}^A & 0 & 0 \\ 0 & V_{sm}^B & 0 \\ 0 & 0 & V_{sm}^C \end{bmatrix} \begin{bmatrix} K_{vs}^A \\ K_{vs}^B \\ K_{vs}^C \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} K_{vs}^P V_{sm}^P \\ K_{vs}^N V_{sm}^N \\ K_{vs}^Z V_{sm}^Z \end{bmatrix} \quad (16)$$

of the form $\mathbf{G}^{ABC} \mathbf{X}^{ABC} = \mathbf{V}^{ABC}$. For data collected at n_t time points, the dimensions of $\mathbf{G}^{ABC}$ and $\mathbf{V}^{ABC}$ are extended to $3n_t \times 3$ and $3n_t \times 1$, respectively. $\mathbf{X}^{ABC}$ can be obtained using the least-squares method, and thus

$$\begin{cases} K_{vs}^A = \mathbf{X}^{ABC}(1) \\ K_{vs}^B = \mathbf{X}^{ABC}(2) \\ K_{vs}^C = \mathbf{X}^{ABC}(3) \end{cases} \quad (17)$$

According to (5), we have

$$\begin{cases} f_{vs}^A = \frac{1}{|K_{vs}^A|} - 1, \delta_{vs}^A = -\arg(K_{vs}^A) \\ f_{vs}^B = \frac{1}{|K_{vs}^B|} - 1, \delta_{vs}^B = -\arg(K_{vs}^B) \\ f_{vs}^C = \frac{1}{|K_{vs}^C|} - 1, \delta_{vs}^C = -\arg(K_{vs}^C) \end{cases} \quad (18)$$

with $\arg$ denoting the phase angle.

In summary, the MM between upstream and downstream VTs can be established and expressed as

$$[\mathbf{f}_{vs}, \boldsymbol{\delta}_{vs}] = H(\mathbf{f}_{vr}, \boldsymbol{\delta}_{vr}). \quad (19)$$

Notably, $\mathbf{f}_{vs} = [f_{vs}^A \ f_{vs}^B \ f_{vs}^C]$, and the definitions of $\boldsymbol{\delta}_{vs}$, $\mathbf{f}_{vr}$, and $\boldsymbol{\delta}_{vr}$ are similar. H denotes the bijective parameter correlation between them to ensure the theoretical foundation for uncertainty propagation. Notably, transmission line parameters remain essentially unchanged over a daily evaluation interval [32]. Accordingly, they are treated as unknown constants in the derivation, with their uncertainty deemed negligible.

Algorithm 1: Parameter Distribution Propagation.

Input : $N(\mu_{c,e,p}, \sigma_{c,e,p}^2)$ (Parameter distribution of calibrated sensor), M (Sample size of MCM)

- 1 **for** $i \leftarrow 1 : M$ **do**
- 2 $[\mathbf{f}_{vr}, \boldsymbol{\delta}_{vr}] \leftarrow \text{Sample}(N(\mu_{c,e,p}, \sigma_{c,e,p}^2))$ // sample from parameter distribution of the calibrated sensor
- 3 $[\mathbf{f}_{vs}(i), \boldsymbol{\delta}_{vs}(i)] = H(\mathbf{f}_{vr}, \boldsymbol{\delta}_{vr})$ // calculate parameters of the uncalibrated sensor based on (19)
- 4 **end**
- 5 $N(\mu_{uc,e,p}, \sigma_{uc,e,p}^2) \leftarrow \text{Fit}(\mathbf{f}_{vs}, \boldsymbol{\delta}_{vs})$ // fit the parameter distribution of the uncalibrated sensor

Output : $N(\mu_{uc,e,p}, \sigma_{uc,e,p}^2)$

B. Propagation of Parameter Distributions

In the proposed MM, the known parameters consist of the sensor measurement data, the inputs are samples drawn from the parameter distribution of the calibrated sensor, and the outputs are the estimated parameters of the uncalibrated sensor. Notably, the parameter of a calibrated sensor follows a normal distribution $N(\mu_c, \sigma_c^2)$, where μ_c denotes the calibration result, and σ_c corresponds to the uncertainty of the reference standard device [33]. Thus, we propose an MCM-based method to propagate parameter distributions between adjacent sensors, and the detailed procedure is outlined in Algorithm 1.

- 1) A sample set $[\mathbf{f}_{vr}, \boldsymbol{\delta}_{vr}]$ is drawn from the parameter distribution $N(\mu_{c,e,p}, \sigma_{c,e,p}^2)$ of the calibrated sensor, where subscript e denotes the type of parameter, i.e., f or δ , and p denotes the phase index.
- 2) Sample set $[\mathbf{f}_{vr}, \boldsymbol{\delta}_{vr}]$ is input into the MM to obtain the corresponding parameter estimates $[\mathbf{f}_{vs}, \boldsymbol{\delta}_{vs}]$ of the uncalibrated sensor.
- 3) The above two procedures are repeated $M = 10000$ times.
- 4) M sets of parameter estimates of the uncalibrated sensor are fitted to obtain the corresponding parameter distribution $N(\mu_{uc,e,p}, \sigma_{uc,e,p}^2)$.

Here, we illustrate the reason for employing the MCM method and its parameter configuration. First, the convergence rate of MCM is $O(M^{-1/2})$, which is independent of dimensionality and underlies its favorable computational complexity [34]. Second, the MCM-approximated log-likelihood converges to the true log-likelihood with suitable M [35]. Relative metrology standard recommends $M \geq 10000$, with larger values needed only for higher order percentiles, though this reduces computational efficiency [24]. As a result, we adopt $M = 10000$ as a balanced and compliant setting, as verified in the numerical analysis section.

By employing the MCM, the parameter distribution of the uncalibrated sensor can be determined, laying the foundation for network-level evaluation of measurement uncertainties.

C. Recursive Framework for Network-Level Evaluation

The recursive workflow for evaluating measurement uncertainties is depicted in Fig. 5. To ensure clarity, each node in the network is uniquely associated with a specific sensor. The process begins at the calibrated node(s) and follows a recursive propagation pathway throughout the network. By propagating

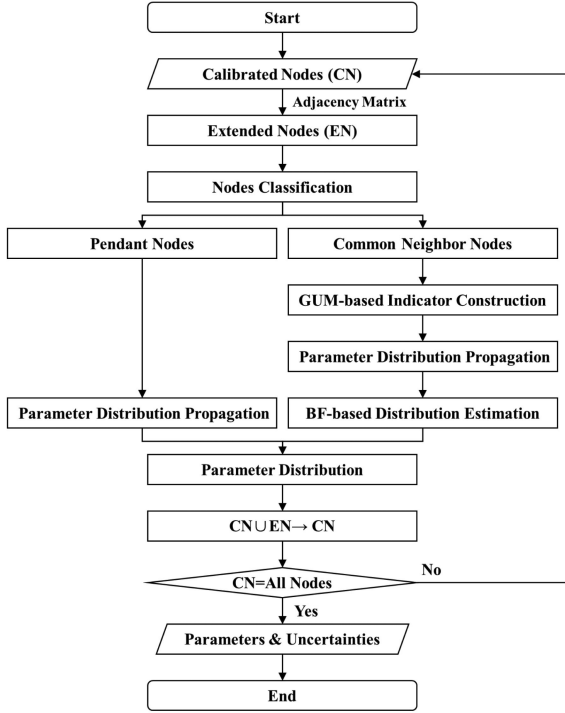


Fig. 5. Recursive workflow for evaluating measurement uncertainties in the sensor network.

parameter distributions, the workflow systematically and sequentially evaluates the parameter distributions of uncalibrated nodes. Once all nodes in the target region have been processed, the measurement uncertainties of all sensors can be obtained.

In the sensor network, those uncalibrated nodes, which are connected to calibrated ones and depicted as extended nodes, can be further classified into pendant nodes and common neighbor nodes. Pendant nodes are defined as nodes that are connected to only a single calibrated node, and the common neighbor nodes are those connected to multiple calibrated nodes. The parameter distribution of a pendant node can be directly evaluated based on the parameter distribution of the single connected calibrated node, using the propagation of parameter distributions. Nevertheless, it is intricate to obtain the parameter distribution of a common neighbor node as it is connected to multiple calibrated nodes. Thus, a hybrid method of GUM and BF with three steps is proposed as follows.

1) GUM-Based Indicator Construction: The calibrated nodes with relatively small uncertainty increments along the propagation pathway are first identified. Given the significant nonlinearity of the MM, a linear approximation is essential for effectively applying uncertainty propagation theory. This approach also aids in qualitatively identifying the factors that contribute to increases in uncertainty.

Here, we give the linearized MM for VTs as an example to quantify the uncertainty increment. For the phase-A network of transmission lines without considering the interphase interference, it can be derived from Kirchhoff's voltage law that

$$\frac{V_{sm}^A}{(1 + f_{vs}^A)e^{j\delta_{vs}^A}} = \left(1 + \frac{Y^A}{2}Z^A\right) \frac{V_{rm}^A}{(1 + f_{vr}^A)e^{j\delta_{vr}^A}} - Z^A I_r^A \quad (20)$$

which can be further reformulated as

$$\frac{1}{1 + f_{vs}^A} = \left(1 + \frac{Y^A}{2}Z^A\right) \frac{V_{rm}^A}{V_{sm}^A} \frac{e^{j(\delta_{vs}^A - \delta_{vr}^A)}}{1 + f_{vr}^A} - \frac{Z^A I_r^A}{V_{sm}^A} e^{j\delta_{vs}^A}. \quad (21)$$

Notably, the voltage phase angle difference across the transmission line and the PDs of VTs are close to 0; moreover, the voltage drop across the transmission line is much smaller than the terminal voltage [13], [36], i.e.,

$$\begin{cases} \frac{V_{rm}^A}{V_{sm}^A} \approx \frac{|V_{rm}^A|}{|V_{sm}^A|} \\ e^{j(\delta_{vs}^A - \delta_{vr}^A)} \approx 1 \\ \frac{Z^A I_r^A}{V_{sm}^A} \approx 0 \end{cases} \quad (22)$$

Thus, (21) can be approximated by

$$1 + f_{vs}^A = \frac{1}{|1 + \frac{Y^A}{2}Z^A|} \frac{|V_{sm}^A|}{|V_{rm}^A|} (1 + f_{vr}^A) + \epsilon \quad (23)$$

where ϵ is a negligible term. Similar linearization assumptions have been discussed in [37]. Based on the uncertainty propagation theory in GUM, the uncertainties of f_{vs}^A and f_{vr}^A satisfy

$$u(f_{vs}^A) = \frac{1}{|1 + \frac{Y^A}{2}Z^A|} \frac{|V_{sm}^A|}{|V_{rm}^A|} \cdot u(f_{vr}^A) = k_A \cdot u(f_{vr}^A) \quad (24)$$

where k_A is the uncertainty propagation coefficient of phase A. This equation indicates that the uncertainty increment is related to the transmission line parameters and the voltage magnitudes of the upstream and downstream nodes. As ZY can be obtained based on (13) and the uncertainties across the three phases are relevant, a coefficient indicator can be constructed as

$$k = k_A + k_B + k_C. \quad (25)$$

By calculating k between the common neighbor node and each calibrated node, n_c calibrated nodes corresponding to the smallest uncertainty increments can be identified. Here, n_c is set to two since a common neighbor node is connected to at least two calibrated nodes.

2) Parameter Distribution Propagation: Following the procedure of propagation of parameter distributions, the parameter distributions of the common neighbor node corresponding to the two selected calibrated nodes can be obtained and denoted as $x_i \sim N(\mu_i, \sigma_i^2) (i = 1, 2)$.

3) BF-Based Distribution Estimation: The results of the preceding step are treated as multiple information sources for the parameter distribution of the common neighbor node. Using the BF method, an estimate of the parameter distribution $\theta \sim N(\mu_0, \sigma_0^2)$ of the common neighbor node is obtained, i.e.,

$$\begin{cases} \mu_0 = \frac{\sum_{i=1}^2 \mu_i / \sigma_i^2}{\sum_{i=1}^2 1 / \sigma_i^2} \\ \sigma_0^2 = \frac{1}{\sum_{i=1}^2 1 / \sigma_i^2} \end{cases} \quad (26)$$

After obtaining the parameter distributions of the extended nodes, these nodes are recursively incorporated into the set of calibrated nodes. If the set of calibrated nodes covers all nodes within the region, the estimate of the parameter (mean of distribution) and uncertainty (half-width of the 95% coverage

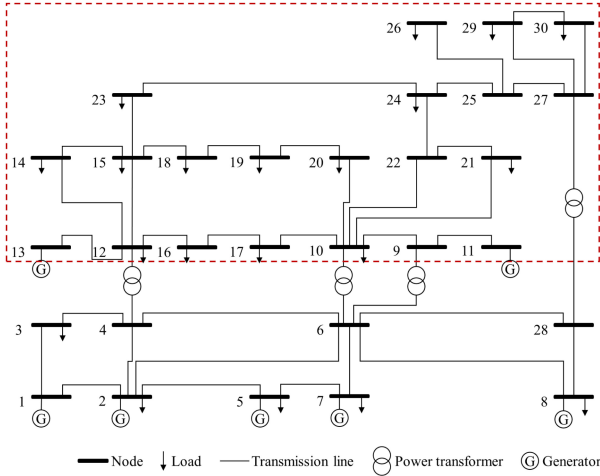


Fig. 6. IEEE 30-node simulation model, where the red dashed box highlights the target sensor network.

interval) for each node can be determined; otherwise, the above procedures will be repeated.

IV. NUMERICAL ANALYSIS

The sensor network composed of VTs in the power system is employed as a case study for numerical analysis. Specifically, the simulation analysis includes three key experiments: the ablation experiment, the calibration-node-type impact experiment, and the sensitivity analysis. These experiments are designed to assess the influence of critical modules on overall performance and to validate the effectiveness of the proposed method in propagating measurement uncertainties. In addition, a real-world analysis is performed to demonstrate the practical applicability of the proposed method.

A. Simulation Analysis

A simulation model of the IEEE 30-node system is built on the Simulink platform, as shown in Fig. 6. It incorporates the essential components required for the normal operation of the power system, including generation units, load models, 220/110 kV transformers, overhead transmission lines, etc. In the 220 kV section, a network comprising 21 nodes is designated as the experimental sensor network. Notably, each node is equipped with three-phase VTs, each transmission line terminal is equipped with three-phase CTs, and each set of VTs/CTs is integrated with a PMU to aggregate the outputs into synchronized phasors. To further simulate the realistic power system operating conditions, the load models are set as recommended in [38].

1) *Experimental Setup*: In the simulation, the node voltages and the line currents at both ends of each transmission line in the target sensor network can be acquired as the ground truth data. Based on the ground truth data, the measurement data is systematically generated as follows. The systematic errors of the sensor network are first initialized by setting the PDs and REs, where random samples are drawn from distributions $N(0, (\frac{0.2\%}{2})^2)$ and $N(0, (\frac{10'}{2})^2)$ according to the limits specified in [39] and [13], respectively. Then, these samples are

superimposed on the ground truth data to emulate measurement data affected by transduction model parameters of CTs/VTs. Furthermore, an additional white noise of 0.1% is introduced to represent the random errors in the measurements. Once a node is designated as the calibrated node, its VT parameter distribution follows

$$\begin{cases} f \sim N\left(f_{\text{set}}, \left(\frac{u(f_{\text{cal}})}{2}\right)^2\right) \\ \delta \sim N\left(\delta_{\text{set}}, \left(\frac{u(\delta_{\text{cal}})}{2}\right)^2\right) \end{cases} \quad (27)$$

where f_{set} and δ_{set} denote the setting RE and PD, and $u(f_{\text{cal}})$ and $u(\delta_{\text{cal}})$ correspond to parameters associated with the uncertainty of the reference standard VT used for calibration. To ensure that the calibration meets the 0.2-class accuracy requirement, the reference standard VT should exhibit an accuracy superior to that of the two accuracy classes of the VT under test [13]. Accordingly, $u(f_{\text{cal}})$ is set to 0.05% and $u(\delta_{\text{cal}})$ to 2.5'.

For experiments involving comparisons among different methods, effect size is used to quantify the differences in their evaluation results. Specifically, effect size is quantified using Cohen's d , which measures the standardized difference between two group means [40] by

$$\begin{cases} d = \frac{\bar{X}_A - \bar{X}_B}{S_p} \\ S_p = \sqrt{\frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{n_A + n_B - 2}} \end{cases} \quad (28)$$

where $\bar{X}$ denotes the sample mean, S denotes the sample standard deviation, n is the sample size, and the subscripts A and B indicate different samples. An absolute effect size greater than 0.8 indicates a substantial difference between the two samples.

2) *Ablation Study*: To validate the effectiveness of the proposed hybrid-based method (HYB-based method), it is compared with its two variants in evaluating measurement uncertainties within the target sensor network. Specifically, the first variant is noted as the GUM-based method, where n_c is set to 1 and the BF step is omitted. The second variant is noted as the BF-based method, where the GUM step is omitted and the BF is applied directly across all calibrated nodes for each common neighbor node.

In this experiment, Node 15 is designated as the calibrated node, and then the aforementioned three methods are utilized to evaluate the measurement uncertainty of each node in parallel. The reason for selecting Node 15 as the calibrated node is that Node 10 becomes a common neighbor node along the uncertainty propagation path, making this setting suitable for distinguishing the performance of the three methods that treat common neighbors differently. To simplify the evaluation results, given the presence of three-phase VTs at each node, the maximum uncertainty is taken as the nodal evaluation uncertainty $u(f)$ and $u(\delta)$. Besides, the root-mean-square error of the three-phase estimated parameters relative to the setting parameters is used as the nodal evaluation bias Δf and $\Delta \delta$.

The detailed results are presented in Tables I and II, with the mean values across nodes 9, 10, and 11 provided at the end of the tables. The results are also visualized in Fig. 7, where the evaluation bias of each node is plotted on the x -axis and the

TABLE I
NODAL EVALUATION UNCERTAINTY OF DIFFERENT METHODS

Nodes	GUM		BF		HYB	
	$u(f)$	$u(\delta)(^{\circ})$	$u(f)$	$u(\delta)(^{\circ})$	$u(f)$	$u(\delta)(^{\circ})$
9	4.94E-04	2.43	2.86E-04	1.40	3.60E-04	1.75
10	4.93E-04	2.43	2.88E-04	1.41	3.65E-04	1.72
11	4.93E-04	2.40	2.78E-04	1.41	3.57E-04	1.72
12	5.14E-04	2.55	5.04E-04	2.55	5.16E-04	2.53
13	5.43E-04	2.58	4.99E-04	2.59	5.13E-04	2.54
14	5.04E-04	2.52	5.15E-04	2.49	4.79E-04	2.49
15	4.90E-04	2.45	4.90E-04	2.45	4.90E-04	2.45
16	5.03E-04	2.48	5.14E-04	2.62	5.16E-04	2.51
17	5.05E-04	2.53	5.26E-04	2.63	5.14E-04	2.50
18	4.99E-04	2.52	5.01E-04	2.44	5.04E-04	2.51
19	5.03E-04	2.43	4.95E-04	2.40	5.14E-04	2.53
20	5.01E-04	2.41	5.01E-04	2.45	5.17E-04	2.45
21	5.43E-04	2.63	4.99E-04	2.35	4.94E-04	2.46
22	5.25E-04	2.52	5.00E-04	2.41	5.06E-04	2.49
23	5.16E-04	2.48	4.99E-04	2.46	4.95E-04	2.47
24	5.13E-04	2.49	5.08E-04	2.39	5.07E-04	2.45
25	5.12E-04	2.41	5.15E-04	2.46	5.01E-04	2.54
26	5.06E-04	2.45	5.04E-04	2.38	4.75E-04	2.50
27	5.04E-04	2.39	5.09E-04	2.55	5.21E-04	2.60
29	5.26E-04	2.41	5.17E-04	2.56	5.09E-04	2.62
30	5.00E-04	2.37	5.16E-04	2.53	5.16E-04	2.56
Mean	4.93E-04	2.42	2.84E-04	1.41	3.61E-04	1.73

The mean values are computed for Nodes 9, 10, and 11.

TABLE II
NODAL EVALUATION BIAS OF DIFFERENT METHODS

Nodes	GUM		BF		HYB	
	Δf	$\Delta \delta(^{\circ})$	Δf	$\Delta \delta(^{\circ})$	Δf	$\Delta \delta(^{\circ})$
9	1.60E-04	0.17	2.48E-04	1.15	1.89E-04	0.84
10	1.25E-04	0.01	2.27E-04	1.01	1.70E-04	0.70
11	2.19E-04	0.35	3.71E-04	1.38	3.06E-04	1.10
12	2.16E-04	0.53	2.19E-04	0.53	2.18E-04	0.51
13	3.34E-04	0.73	3.49E-04	0.71	3.35E-04	0.69
14	1.98E-04	0.62	2.20E-04	0.55	2.13E-04	0.58
15	2.75E-05	0.01	2.75E-05	0.01	2.75E-05	0.01
16	4.56E-04	1.65	4.57E-04	1.65	4.51E-04	1.66
17	4.35E-04	1.84	4.38E-04	1.84	4.27E-04	1.82
18	7.36E-06	0.04	6.05E-06	0.01	4.63E-07	0.04
19	1.42E-05	0.22	1.40E-05	0.18	3.62E-06	0.20
20	5.27E-05	0.43	5.85E-05	0.49	4.64E-05	0.51
21	1.82E-04	1.19	1.81E-04	1.18	1.97E-04	1.21
22	3.35E-04	1.40	3.33E-04	1.46	3.46E-04	1.48
23	7.46E-05	0.63	7.06E-05	0.62	8.01E-05	0.62
24	2.13E-04	1.27	2.12E-04	1.32	2.28E-04	1.31
25	2.27E-04	1.00	2.19E-04	1.02	2.40E-04	1.02
26	1.51E-04	1.00	1.51E-04	1.03	1.68E-04	1.00
27	2.04E-04	1.62	1.94E-04	1.64	2.19E-04	1.61
29	2.26E-04	1.62	2.18E-04	1.63	2.33E-04	1.63
30	2.53E-04	2.14	2.48E-04	2.25	2.65E-04	2.21
Mean	1.96E-04	0.88	2.12E-04	1.03	2.08E-04	0.99

The mean values are computed for Nodes 9, 10, and 11.

corresponding evaluation uncertainty on the y -axis. The results indicate that the evaluation results of the three methods exhibit notable discrepancies across nodes 9, 10, and 11. This is because node 10 is a common neighbor node and nodes 9 and 11 are its extended nodes, the different mechanisms for common neighbor nodes among the three methods lead to substantial variations in the results. An additional table is added to quantify the differences between the GUM- and BF-based methods relative to the HYB-based method using effect size, with a specific focus on Nodes 9, 10, and 11, as shown in Table III. The mean values and effect sizes indicate that, regarding uncertainty, $BF < HYB < GUM$, and regarding bias, $GUM < HYB < BF$. The

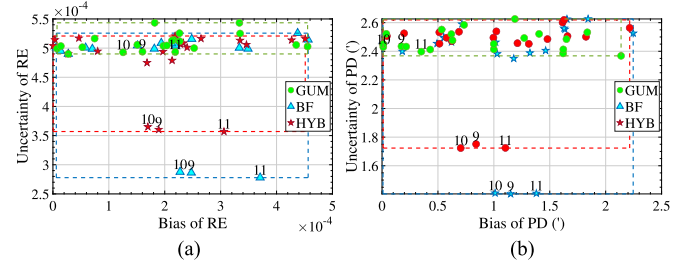


Fig. 7. Scatter plots of evaluation bias versus evaluation uncertainty. The results achieved by each method are highlighted by dashed boxes in corresponding colors. Nodes 9, 10, and 11 are highlighted due to significant differences in the evaluation results of the three methods. (a) Uncertainty and bias of RE. (b) Uncertainty and bias of PD.

TABLE III
EFFECT SIZES OF RESULTS OBTAINED BY DIFFERENT METHODS

Uncertainty/Bias	GUM versus HYB		BF versus HYB	
	f	δ	f	δ
u	+45.45	+37.93	-16.26	-28.84
Δ	-0.87	-3.76	+0.80	+1.53

The effective sizes are computed for Nodes 9, 10, and 11, using the results of the HYB-based method as the reference sample. An absolute effect size greater than 0.8 indicates a substantial difference between the two samples.

GUM-based method selects the calibrated nodes with the minimum uncertainty increment, disregarding the information from other calibrated nodes and thus leading to larger evaluation uncertainty. While the BF-based method achieves lower uncertainty through multisource fusion, its performance is sensitive to source quality, and data disturbances may introduce larger evaluation biases. By contrast, the HYB-based method offers a balanced performance, which achieves tolerable uncertainty while maintaining high evaluation accuracy. This is a property that supports long-distance propagation of measurement uncertainties within sensor networks.

3) Impact of Node Types: This experiment is conducted to analyze whether the evaluated uncertainty is influenced by the types of calibrated nodes. Based on previous studies on nodal connectivity strength and link characteristics in networks [41], [42], among the 21 nodes of the target sensor network, node 13 is classified as weakly connected, node 10 as strongly connected, and node 24 as moderately connected. Using each of these three nodes as the calibrated node, the HYB-based method is employed to evaluate the measurement uncertainties within the target sensor network.

The detailed results are presented in Tables IV and V. Topology diagrams are generated to visualize the propagation of parameter distributions, with each node represented by a circle, as illustrated in Fig. 8. The left half of each circle indicates the nodal evaluation bias, the right half reflects the nodal evaluation uncertainty, and the colors are used to signify the magnitudes. The maximum uncertainty and bias during the propagation process are indicated in the legend. The results reveal that the maximum uncertainty is independent of the connectivity strength of calibrated nodes, indicating that the method is unaffected by the types of calibrated nodes.

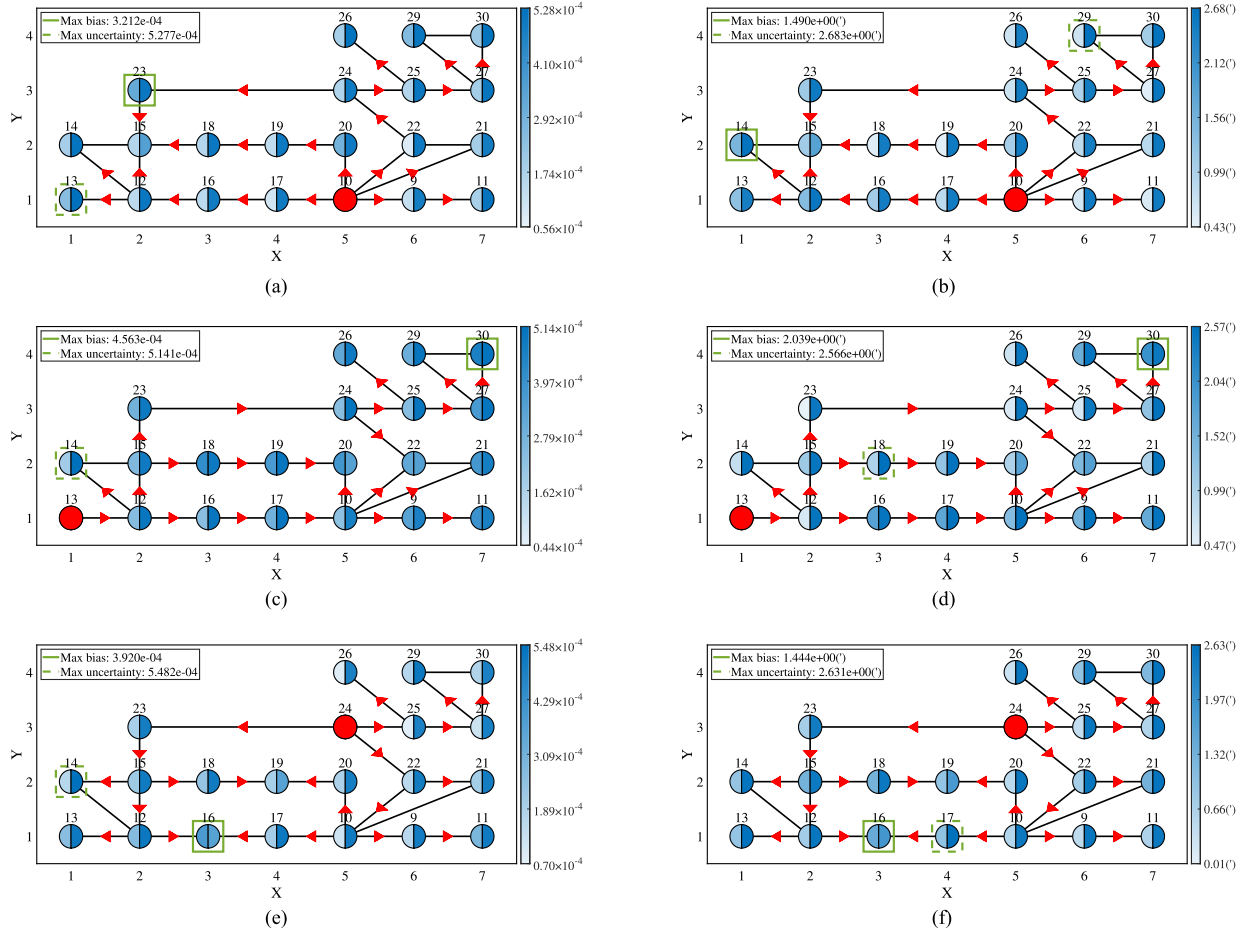


Fig. 8. Evaluation results under different settings of the calibrated node (red circle), where the red arrows indicate the pathways of parameter distribution propagation. The node within a green solid square exhibits the largest evaluation bias, while the node within a green dashed square exhibits the largest evaluation uncertainty. (a) Evaluation uncertainty and bias of RE. (strongly). (b) Evaluation uncertainty and bias of PD. (strongly). (c) Evaluation uncertainty and bias of RE. (weakly). (d) Evaluation uncertainty and bias of PD. (weakly). (e) Evaluation uncertainty and bias of RE. (moderately). (f) Evaluation uncertainty and bias of PD. (moderately).

TABLE IV
NODAL EVALUATION UNCERTAINTY OF DIFFERENT CONNECTIVITY STRENGTHS OF CALIBRATED NODES

Nodes	Strongly		Weakly		Moderately	
	$u(f)$	$u(\delta)(^\circ)$	$u(f)$	$u(\delta)(^\circ)$	$u(f)$	$u(\delta)(^\circ)$
9	4.86E-04	2.54	4.59E-04	2.55	5.08E-04	2.54
10	4.90E-04	2.45	4.67E-04	2.54	5.15E-04	2.51
11	5.21E-04	2.51	4.61E-04	2.52	5.16E-04	2.50
12	5.07E-04	2.58	4.99E-04	2.52	5.25E-04	2.52
13	5.28E-04	2.46	4.90E-04	2.45	5.21E-04	2.62
14	5.14E-04	2.64	5.14E-04	2.54	5.48E-04	2.56
15	3.55E-04	1.83	4.83E-04	2.49	5.15E-04	2.48
16	4.94E-04	2.50	4.97E-04	2.51	3.68E-04	1.86
17	4.92E-04	2.51	4.98E-04	2.54	5.21E-04	2.63
18	5.07E-04	2.49	4.89E-04	2.57	5.15E-04	2.50
19	5.02E-04	2.54	4.97E-04	2.54	3.64E-04	1.79
20	4.96E-04	2.53	3.37E-04	1.76	5.03E-04	2.53
21	4.87E-04	2.57	4.69E-04	2.55	5.07E-04	2.59
22	4.96E-04	2.48	3.43E-04	1.80	5.07E-04	2.54
23	5.08E-04	2.54	4.88E-04	2.48	5.05E-04	2.43
24	4.98E-04	2.51	5.04E-04	2.50	4.90E-04	2.45
25	5.07E-04	2.49	4.99E-04	2.47	4.91E-04	2.53
26	5.25E-04	2.47	4.85E-04	2.48	4.91E-04	2.54
27	5.11E-04	2.61	5.02E-04	2.54	5.00E-04	2.56
29	5.09E-04	2.68	4.99E-04	2.51	5.25E-04	2.58
30	5.24E-04	2.65	5.06E-04	2.47	4.89E-04	2.58

TABLE V
NODAL EVALUATION BIAS OF DIFFERENT CONNECTIVITY STRENGTHS OF CALIBRATED NODES

Nodes	Strongly		Weakly		Moderately	
	Δf	$\Delta \delta(^\circ)$	Δf	$\Delta \delta(^\circ)$	Δf	$\Delta \delta(^\circ)$
9	1.23E-04	0.61	3.34E-04	1.49	1.63E-04	0.45
10	5.62E-05	0.43	2.22E-04	1.34	1.17E-04	0.57
11	1.79E-04	0.55	3.95E-04	1.64	2.91E-04	0.64
12	1.41E-04	1.37	2.42E-04	0.61	2.72E-04	0.78
13	2.48E-04	1.29	4.42E-05	0.88	2.62E-04	0.92
14	1.88E-04	1.49	1.84E-04	0.77	1.69E-04	0.93
15	1.57E-04	1.10	2.51E-04	1.12	2.19E-04	1.29
16	1.42E-04	1.10	2.32E-04	1.77	3.92E-04	1.44
17	9.91E-05	0.84	2.81E-04	1.71	2.05E-04	0.98
18	1.58E-04	0.49	4.11E-04	0.98	2.34E-04	1.30
19	1.35E-04	0.45	3.65E-04	1.22	2.00E-04	0.92
20	2.91E-04	1.05	3.80E-04	0.99	1.90E-04	0.51
21	1.92E-04	0.87	3.74E-04	0.98	2.07E-04	1.31
22	7.71E-05	0.82	3.15E-04	1.40	1.57E-04	0.73
23	3.21E-04	1.12	2.82E-04	0.47	1.98E-04	0.74
24	1.66E-04	0.69	2.34E-04	0.59	6.95E-05	0.01
25	1.59E-04	0.97	2.37E-04	0.59	7.86E-05	0.39
26	1.82E-04	0.67	3.14E-04	0.69	8.21E-05	0.33
27	1.92E-04	0.50	3.68E-04	1.23	1.23E-04	0.61
29	2.59E-04	0.67	2.92E-04	1.57	1.71E-04	0.96
30	2.10E-04	1.08	4.56E-04	2.04	1.79E-04	1.18

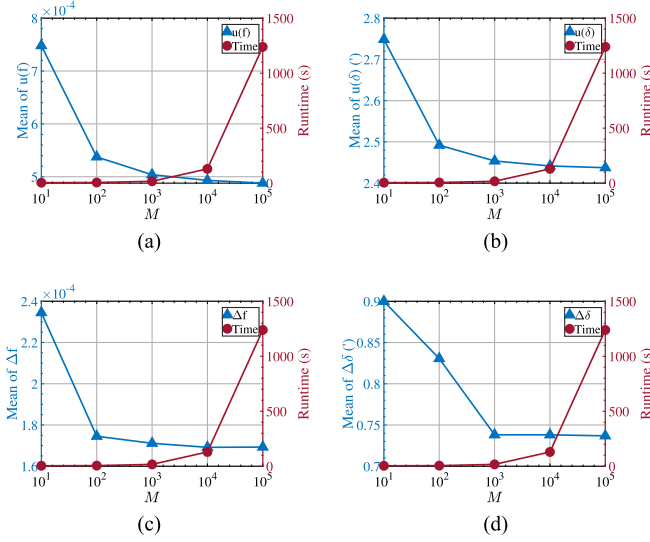


Fig. 9. Evaluation results and runtime under different settings of M . (a) Uncertainty of RE and runtime. (b) Uncertainty of PD and runtime. (c) Bias of RE and runtime. (d) Bias of PD and runtime.

4) Sensitivity Analysis: To verify the appropriateness of the choice of the Monte Carlo sample size M , the sensitivity analysis is conducted. Once Node 24 is used as the calibrated node, the HYB-based method is applied with different values of M to evaluate the measurement uncertainties within the target sensor network. The mean values of the evaluation results across all nodes are calculated, and the variations of both the mean values and the runtime with respect to M are shown in Fig. 9. The results show that as M increases, the outcomes gradually converge and stabilize, while the runtime increases exponentially. Furthermore, when M exceeds 1000, the evaluation results are essentially stable, but the runtime rises sharply for $M > 10,000$. Hence, $M = 10,000$ represents a suitable tradeoff between convergence and computational complexity, ensuring sufficient robustness of the method.

Simulation results confirm that the proposed method achieves high evaluation accuracy while ensuring tolerable uncertainty. In the context of power system sensor networks, the method effectively enables long-distance propagation of both parameters and uncertainties. Notably, the evaluation performance remains unaffected by the connectivity strength of calibrated nodes.

B. Real-World Analysis

To further evaluate the effectiveness of the proposed method for assessing measurement uncertainties in real-world sensor networks, two 220 kV substations in Jilin, China, are selected and designated as Substations S and R . The voltage and current measurement data from these substations are collected for subsequent analysis. Notably, the measurements are collected from PMUs, which are connected downstream of VTs/CTs. We calibrated the PMU biases and reduced their uncertainty to below 0.01%, making their impact on the uncertainty evaluation of VT negligible.

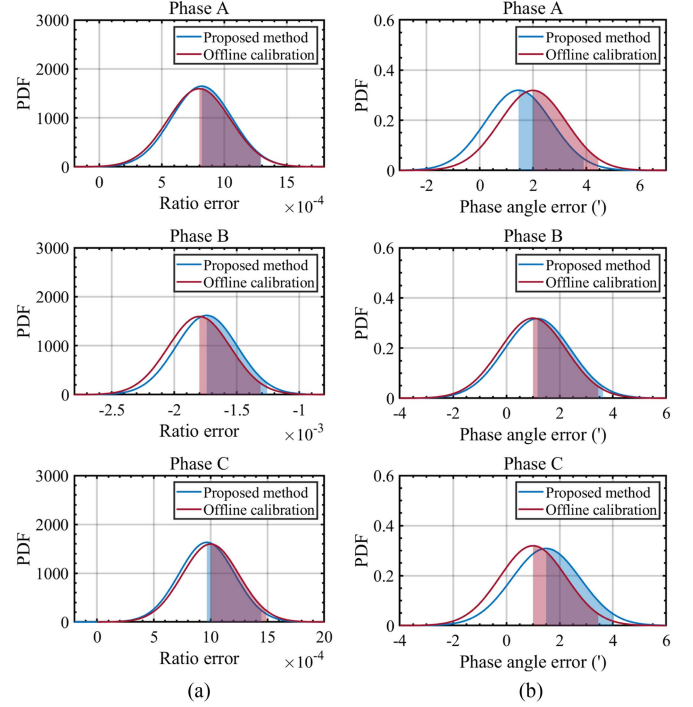


Fig. 10. Comparison between the evaluated and calibrated results of VTs at substation R . The shaded area represents the range encompassed by the estimated parameter and the associated uncertainty. (a) RE distribution at substation R . (b) PD distribution at substation R .

Given that the proposed method adopts a recursive mechanism, verifying the evaluation accuracy for a single layer of uncertainty propagation is sufficient to ensure the method's reliability across the entire network. As such, the selection of two substations for practical validation is considered representative. Specifically, the VTs at both substations have undergone offline calibration, and their parameters and uncertainties are determined beforehand. Then, Substation S is designated as the calibrated node, and the proposed method is applied to estimate the parameter distributions at Substation R .

The estimated parameter distributions are compared with the calibrated results of Substation R , as illustrated in Fig. 10. In the figure, the mean of the PDF is used as the estimated parameter, the half-width of the 95% coverage interval is used to quantify the uncertainty, and the shaded area in the figure reflects the encompassed range. As can be observed, there is a significant overlap between the evaluated and calibrated shaded areas, demonstrating strong consistency in both parameters and uncertainties. This high degree of agreement confirms the practical applicability and effectiveness of the proposed method in real-world scenarios.

V. DISCUSSION

The results presented in Tables IV and V and Fig. 8 indicate that the evaluated uncertainty and bias exhibit certain patterns during the propagation process. Specifically, the magnitude of the uncertainty consistently remains tolerable, with the uncertainties of extended nodes remaining nearly identical to those of the calibrated nodes. This phenomenon can be attributed to the

TABLE VI
RELATIONSHIP BETWEEN THE UNCERTAINTY PROPAGATION
COEFFICIENT AND TRANSMISSION LINE LENGTH

Line length (km)	Propagation coefficient
60	1.002
180	1.016
300	1.046

The transmission line parameters are specified as resistance $R = 0.1 \Omega/\text{km}$, inductance $L = 1 \times 10^{-3} \text{ H/km}$, and capacitance $C = 1 \times 10^{-8} \text{ F/km}$.

precision and intrinsic characteristics of the constructed MM. Using the parameters of a typical 220 kV overhead transmission line operating at 50 Hz as an example [43], the relationship between the uncertainty propagation coefficient (as derived in (24), the upstream and downstream voltage magnitudes are both approximately 220 kV and assumed to have a ratio of 1) and the transmission line length is presented in Table VI. The small magnitude of the uncertainty propagation coefficient further supports this observation. This phenomenon may differ in other sensor networks. The proposed method is designed to recursively evaluate uncertainty and quantify its growth. Whether the resulting uncertainty levels are acceptable depends on domain-specific operational criteria, which vary across countries, utilities, and sensor-quality classes.

In contrast to uncertainty, evaluation bias tends to increase as it propagates. Factors, such as time synchronization errors, PMU quantization errors, and similar issues, can introduce disturbances into the measurement data. As the propagation distance increases, the cumulative effects of data disturbances become more significant, inevitably leading to increased biases. Further investigation is required to suppress incremental biases using techniques, such as data smoothing [15].

To the best of authors' knowledge, this work is the first to advance the view that uncertainty quantification is as important as bias calibration. Our work focuses on using the VT networks as a representative case study to investigate the practical feasibility of the proposed method. Benefiting from its recursive mechanism, the proposed method exhibits strong adaptability, enabling automatic reconfiguration of propagation pathways in response to topology changes. For sensor networks lacking precise physical models or redundant data, uncertainty propagation remains feasible and informative. Evaluation accuracy may be constrained due to challenges in parameter identification, yet the proposed method can still provide useful information to support sensing performance. From the perspective of technological advancements, i.e., the development trend toward large-scale [44], highly connected [45], edge-enabled architectures [46], and the increasing demand for reliability assessment in sensor networks, there is promising potential for method deployment.

V. CONCLUSION

In this study, we propose a recursive framework for online evaluation of measurement uncertainties in sensor networks. An MM is established to characterize the relationships between interdevice transduction model parameters, and the MCM is

employed to propagate parameter distributions across inter-connected devices. Building on this, we propose a recursive workflow that begins from calibrated node(s) and recursively extends through uncalibrated units, enabling the quantification of parameters and uncertainties across the entire sensor network. Specifically, the uncertainty propagation theory is incorporated to identify calibrated nodes with the minimum uncertainty increment, while the BF is applied to estimate the parameter distributions of extended nodes.

We focus on VTs in the smart grid and validate the proposed method through both simulations and real-world analyses. The results demonstrate that it is both theoretically sound and practically applicable. From the perspective of IoT trust management frameworks, such as StackTrust [6], the proposed method can enhance system trustworthiness by evaluating measurement uncertainty under realistic operating conditions. Furthermore, it provides a solid foundation for advancing research on the optimal selection of calibration nodes, offering valuable theoretical guidance and decision-making support for the operation and maintenance of sensor networks. Looking ahead, this method and its derivative studies could be extended to critical sensor networks, such as charging infrastructures, optical monitoring systems, and industrial control networks, thereby promoting broader deployment and deeper integration within the smart grid, industrial IoT, and related domains.

V. ACKNOWLEDGMENT

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