A Radial Space Division Based Many-Objective Optimization Evolutionary Algorithm

Cheng He

School of Automation,

Huazhong University of Science and Technology,

Wuhan 430074, P. R. China

chenghehust@gmail.com

Outline

- Many-Objective Optimization
- Radial Projection
- The Proposed AREA
- Experimental Results
- Conclusion

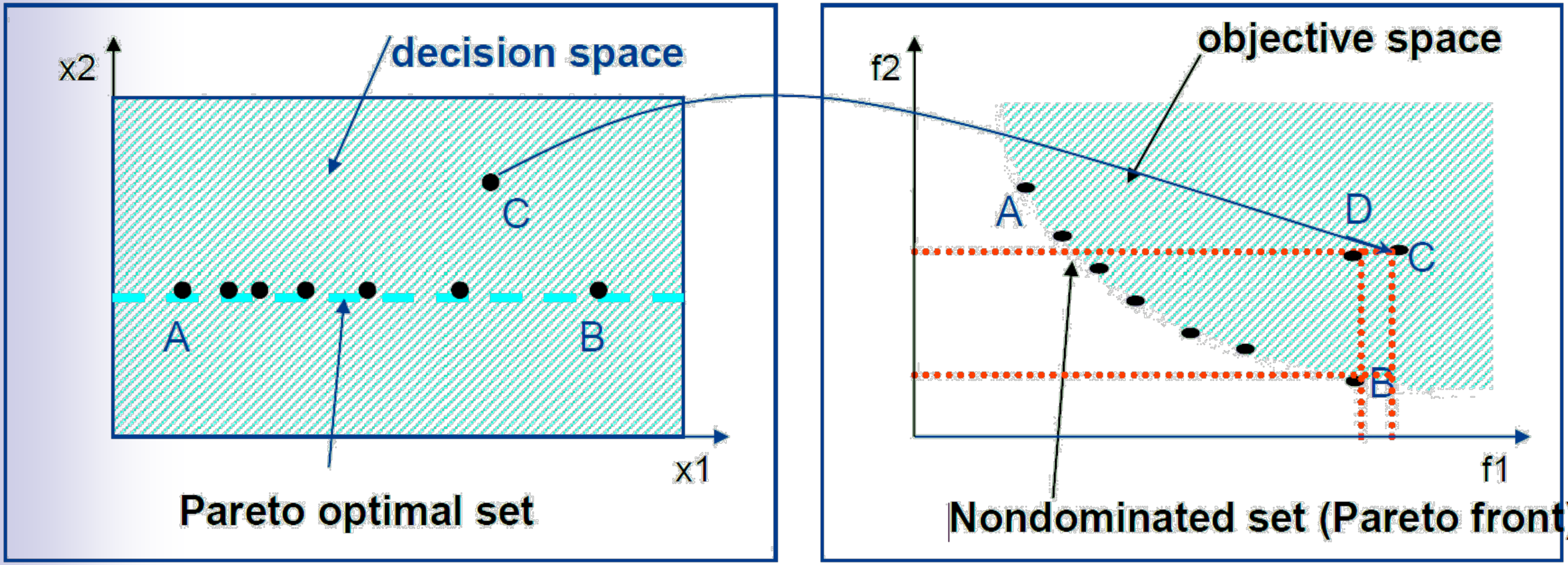
Many-Objective Optimization

- Formulation

$$\text{Minimize } F(x) = (f_1(x), f_2(x), \ldots, f_m(x))$$
$$\text{subject to } \mathbf{x} \in X.$$

PS: Pareto optimal set

PF: Pareto optimal front



Many-Objective Optimization

- Difficulties for solving MaOPs
  - The loss of selection pressure--- unable to distinguish non-dominated solutions
  - Diversity maintenance--- “curse of dimensionality”

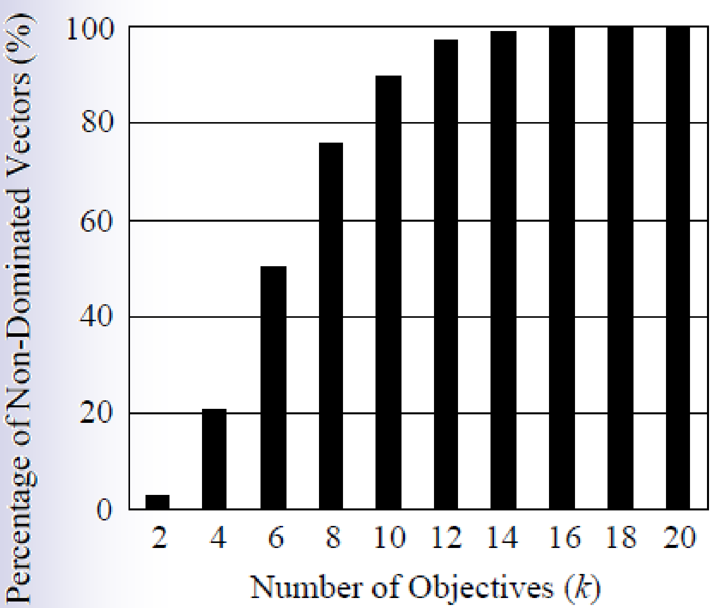
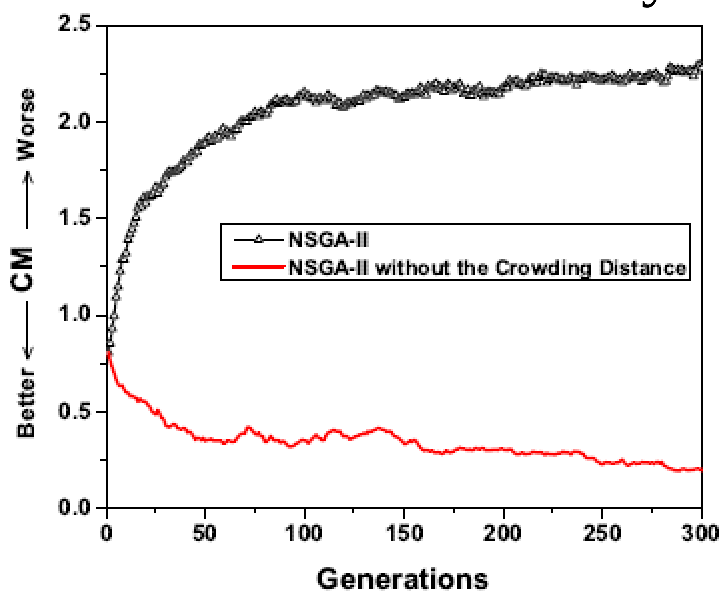


Fig. 1. Average percentage of non-dominated vectors among 200 vectors that are randomly generated in the k-dimensional unit hypercube $[0, 1]^k$.



Evolutionary trajectories of the convergence metric (CM) for a run of the original NSGA-II and the modified NSGA-II without the density estimation procedure on the 10-objective DTLZ2.

Ishibuchi H, Tsukamoto N, Nojima Y. Evolutionary many-objective optimization: A short review. IEEE congress on evolutionary computation. 2008: 2419-2426.

Li M, Yang S, Liu X. Shift-based density estimation for Pareto-based algorithms in many-objective optimization. IEEE Transactions on Evolutionary Computation, 2014, 18(3): 348-365.

Many-Objective Optimization

- Existing Many-Objective Optimization Algorithms
  - Pareto-based MaOEAs
    1. With modified dominance relationship, e.g., fuzzy dominance, ϵ −dominance, and preference rank order.
    2. With additional convergence-related criterion, e.g., the concept of "knee point" in KnEA and the "grid-dominance" in GrEA.
  - Reference-based MaOEAs
    1. Decomposition-based MaOEAs, e.g., MOEA/D, MOEA/D-DU, and RVEA.
    2. Preference based MaOEAs, PICEAg and NSGA-III.
  - Indicator-based MaOEAs
    - HypE, IBEA, and MOMBI-II
  - Others like Two_Arc2 and MOEA-DVA

Many-Objective Optimization

- Existing Diversity Maintenance Strategies
  - Crowding degree estimation based on the distances to neighboring solutions, e.g., crowding distance computation in NSGA-II and weighted distance computation in KnEA.
  - Region division based strategies, e.g, the region-based approach in PESAII and the grid division in GrEA.
  - Reference information based strategies, e.g., reference points in NSGA-III and reference vectors in RVEA.

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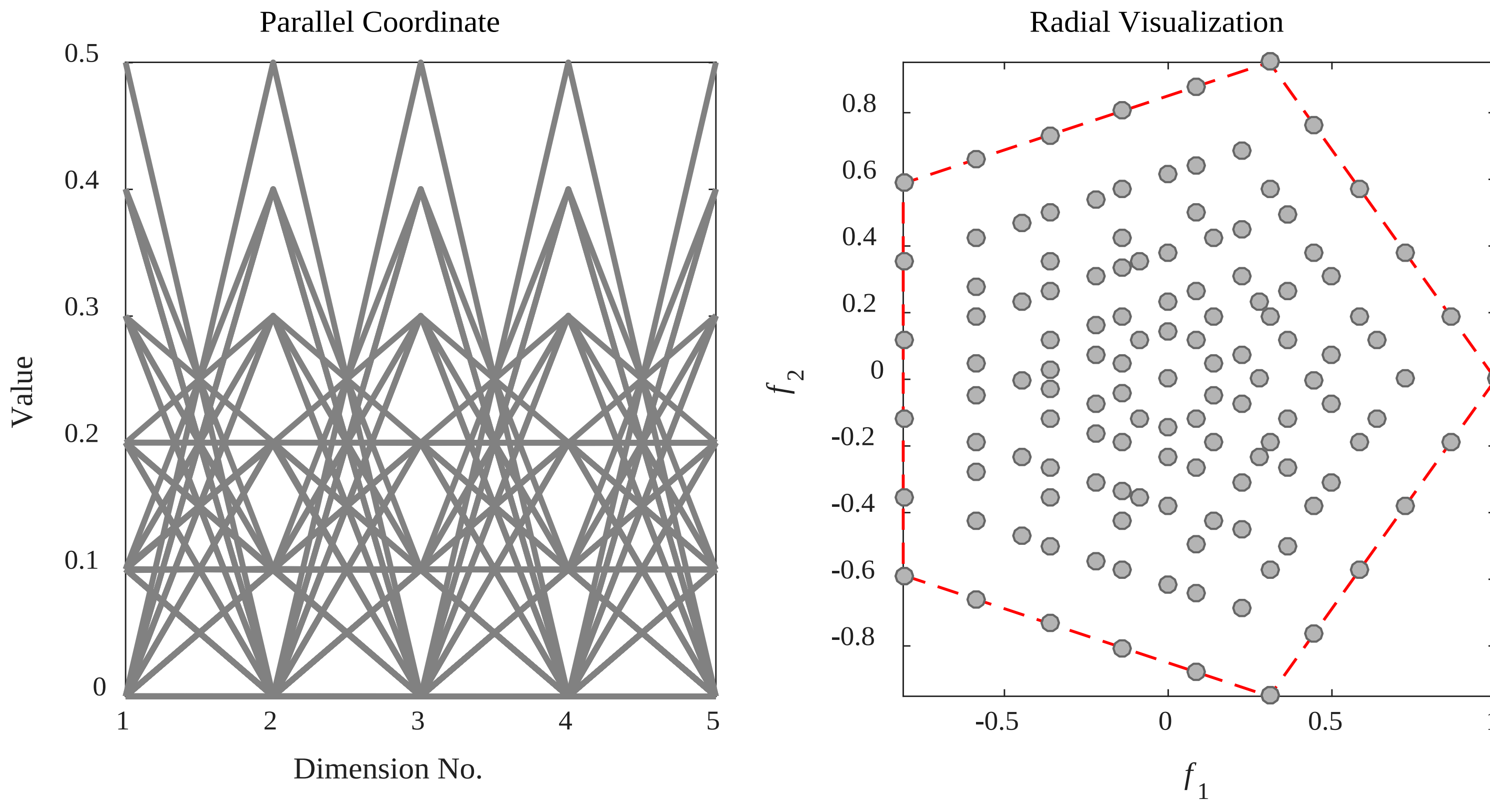
Radial Projection



Radial Projection

- Distribution Reflection

Suppose $X_1, X_2 \in \Omega^m$ are two points in the m-dimensional space, and their coordinates in the radial place are Y_1 and Y_2, respectively.

$$E_m = \| X_1 - X_2 \|, E_2 = \| Y_1 - Y_2 \|$$

$$\begin{aligned} E_2 &= \| (X_1 W_1 (X_1 I)^{-1}, X_1 W_2 (X_1 I)^{-1}) - (X_2 W_1 (X_2 I)^{-1}, X_2 W_2 (X_2 I)^{-1}) \| \\ &= \| (((X_1 I)^{-1} X_1 - (X_2 I)^{-1} X_2) W_1, ((X_1 I)^{-1} X_1 - (X_2 I)^{-1} X_2) W_2) \|. \end{aligned}$$

Let $L = (X_1 I)^{-1} X_1 - (X_2 I)^{-1} X_2 = (l_1, l_2, \ldots, l_m)$, then

$$\begin{aligned} E_2 &= \| (LW_1, LW_2) \| = \sqrt{(LW_1, LW_2)((LW_1)^T, (LW_2)^T)^T} \\ &= \sqrt{LW_1 W_1^T L^T + LW_2 W_2^T L^T} \\ &= \sqrt{L(W_1 W_1^T + W_2 W_2^T) L^T}. \end{aligned}$$

Radial Projection

Assume $R = W_1 W_1^T + W_2 W_2^T$, then

$$R = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{im-1} & a_{1m} \\ & & \cdots & & \\ a_{i1} & \cdots & a_{ij} & \ldots & a_{im} \\ & & \cdots & & \\ a_{m1} & a_{m2} & \ldots & a_{mm-1} & a_{mm} \end{bmatrix},$$

where $a_{ij} = \cos(\theta_i)\cos(\theta_j) + \sin(\theta_i)\sin(\theta_i) = \cos(\theta_i - \theta_j).$

Hence,

$$E_2 = \sqrt{LRL^T} = \sqrt{\sum_{j=1}^{m} l_j \left(\sum_{i=1}^{m} l_i \cos(\theta_i - \theta_j) \right)}$$

$$= \sqrt{\underbrace{\sum_{j=1}^{m} l_j^2}_{\text{Part I}} + \underbrace{\sum_{i \neq j}^{i,j \in [1,m]} l_i l_j \cos(\theta_i - \theta_j)}_{\text{Part II}}}.$$

Radial Projection

"Part I"

$$\sum_{j=1}^{m} l_j^2 = (X_1 I)^{-2} X_1 X_1^T - (X_1 I)^{-1}(X_2 I)^{-1} X_1 X_2^T - (X_1 I)^{-1}(X_2 I)^{-1} X_2 X_1^T + (X_2 I)^{-2} X_2 X_2^T$$

is linearly approximate to

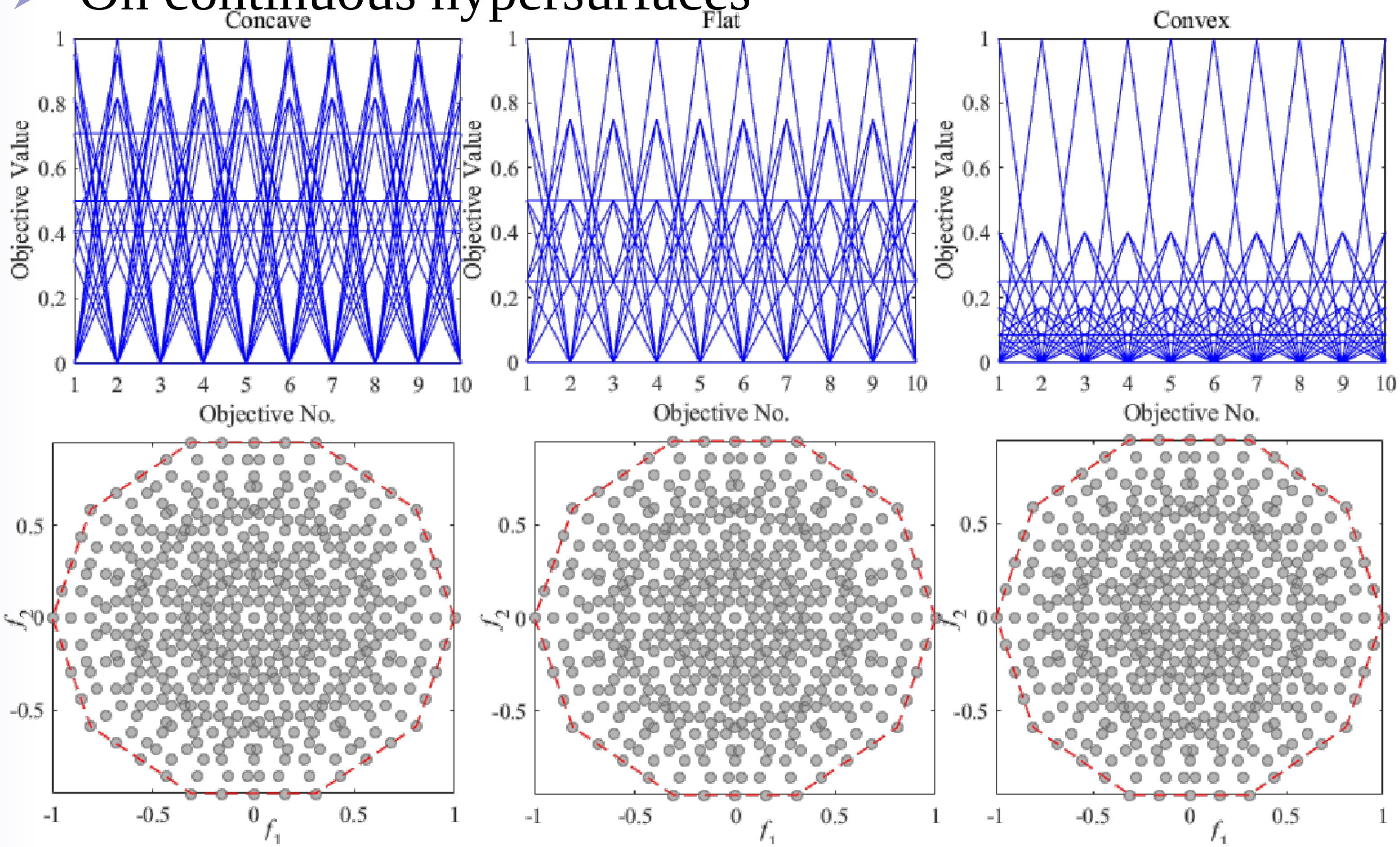
$$E_m = X_1 X_1^T - X_1 X_2^T - X_2 X_1^T + X_2 X_2^T.$$

"Part II" effects less on E_2 than "Part I" as $\cos(\theta_i - \theta_j) < 1$.

$$E_2 = \sqrt{\underbrace{\sum_{j=1}^{m} l_j^2}_{\text{Part I}} + \underbrace{\sum_{\substack{i,j\in[1,m] \\ i\neq j}} l_i l_j \cos(\theta_i - \theta_j)}_{\text{Part II}}}$$

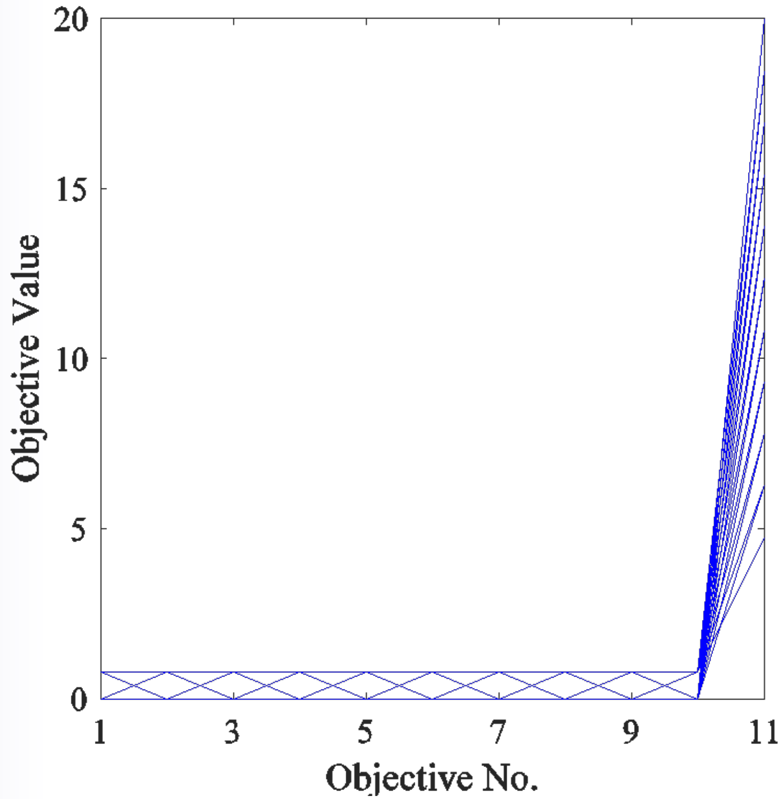
Radial Projection

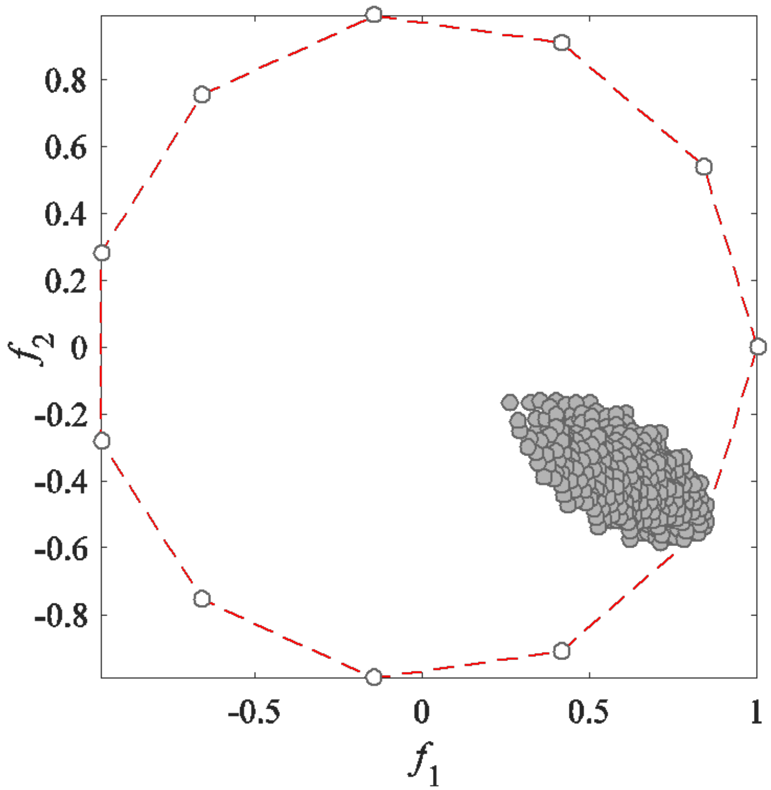
- Analysis
  - On continuous hypersurfaces



Radial Projection

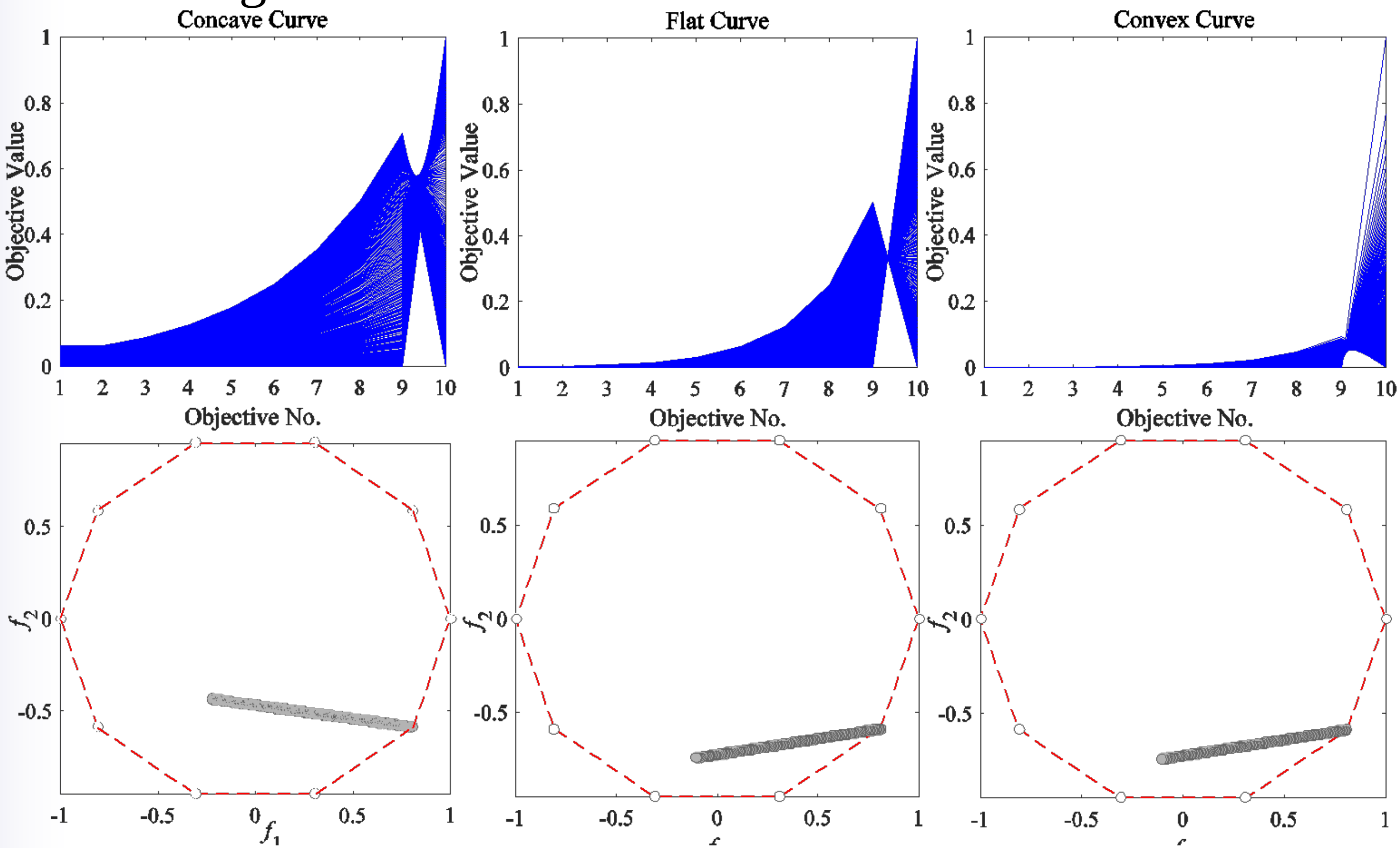
- On discontinuous hypersurface





Radial Projection

- On degenerated curves



Radial Projection

- Conclusion
  - The relationship of the crowding degree information between the original space and the radial space

    (diversity information remains)
  - The missing of the curvature information of the Pareto front (convergence degree is lost)

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- The Proposed AREA
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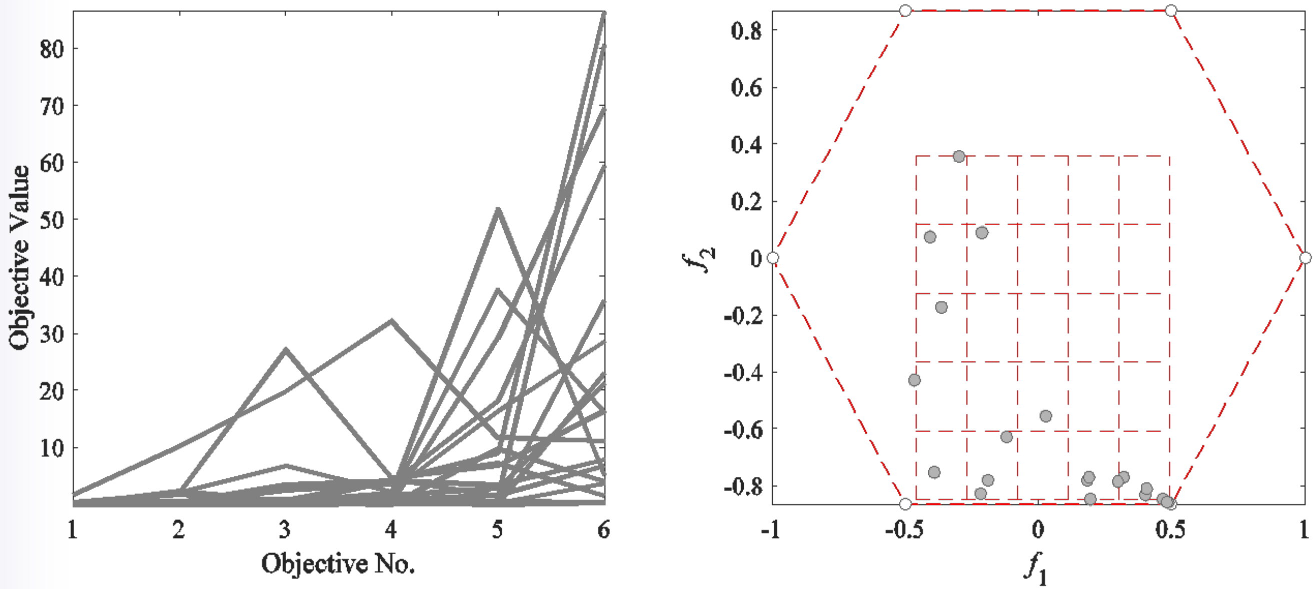
The Proposed AREA

- General Idea
  - Adopt a similar framework as NSGA-II
  - Radial space division for diversity enhancement
  - Diversity-first-convergence-second[1]



[1] Jiang S, Yang S. A strength Pareto evolutionary algorithm based on reference direction for multi-objective and many-objective optimization. 2016.

The Proposed AREA

- General Framework

Algorithm 1 General Framework of AREA

Input:

N (size of population), r (penalty).

$P_0 \leftarrow Initialization(N)$

while termination criterion not fulfilled **do**

  $G \leftarrow Radial\_Grid(P_0, N))$

  $P \leftarrow Mating\_Selection(P_0, G)$

  $P' \leftarrow Variation(P, N)$

  $P_0 \leftarrow Envionmental\_Selection(P' \cup P, N, r)$

end

Return: P_0

The Proposed AREA

- Mating Selection
  - Crowding degree:

The number of solutions in the same rectangle

$$Crowd(G_s) = |S|.$$

  - Convergence degree:

The distance to ideal point

$$Con(F_i) = \left\| \frac{F_i - F_{min}}{F_{max} - F_{min}} \right\|.$$

Algorithm 3 *Mating_Selection*

Input:
P (population), G (rectangle labels).
Output:
H (mating pool population)

$Q, H \leftarrow \emptyset$
$Crowd \leftarrow$ Calculate the crowding degrees of solutions in P according to Eq. (4)
$Con \leftarrow$ Calculate the convergence degrees of solutions in P according to Eq. (5)
while $|Q| < |P|$ **do**
  randomly select two rectangles a and b from G
  if $Crowd(a) < Crowd(b)$ **then**
    $Q \leftarrow Q \cup \{a\}$
  else
    $Q \leftarrow Q \cup \{b\}$
  end
end
for $i \leftarrow 1 : |P|$ **do**
  $S \leftarrow$ Find the solutions in the ith rectangle in Q
  randomly select two solutions w and v from S
  if $Con(w) < Con(v)$ **then**
    $H \leftarrow H \cup \{w\}$
  else
    $H \leftarrow H \cup \{v\}$
  end
end
Return: H

The Proposed AREA

- Environmental Selection
  - Extreme solutions
  - Least crowded rectangles
  - Better fitness

$$Fit(X,Q) = Con(X) \cdot r \cdot m - \min \| Y(X) - Y(Q) \|$$

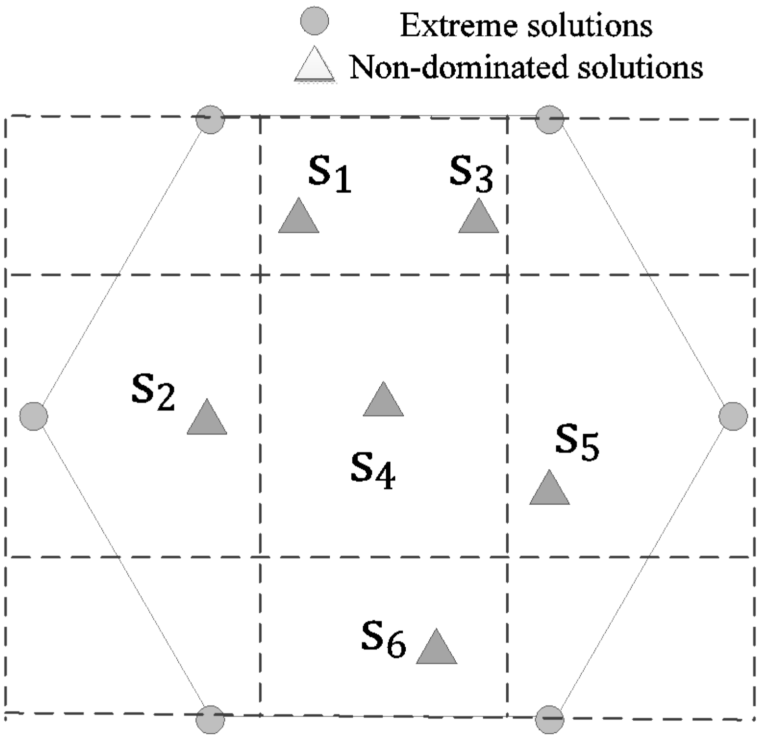
Algorithm 4 *Environmental_Selection*

Input:
P (population), N (population size), r (penalty).

$Q \leftarrow \emptyset$
$(F_1, F_2, \ldots) \leftarrow$ Non-dominated-sort(P)
$P \leftarrow F_1 \cup F_2 \cup \cdots \cup F_i$
/*$|F_1 \cup \cdots \cup F_{i-1}| < N$, $|F_1 \cup \cdots \cup F_i| \geq N$*/
$[Y, G] \leftarrow Radial\_Grid(P, N)$
$C \leftarrow 0$ /*Initialize the crowding degree*/
$Q \leftarrow$ Select the extreme solutions in P
$C_Q \leftarrow 1$ /*Update the crowding degree of the rectangles with extreme solutions*/
$I \leftarrow \{I_1, \ldots, I_j\}$ /*merge solutions in rectangle j into set I_j based on G*/
while $Q < N$ **do**
    $K \leftarrow \arg\min C$/*find the set of solutions in the least crowded rectangles*/
    $Fit(K) \leftarrow$ Calculate the fitness values of solutions in K by Eq. (6)
    $P_q \leftarrow \arg_{q \in K} \min Fit(K)$
    $Q \leftarrow Q \cup \{P_q\}$
    $P \leftarrow P \backslash \{P_q\}$/*delete the selected solution*/
    $C_q \leftarrow C_q + 1$/*update the crowding degree of the selected rectangle*/
end

The Proposed AREA

- Example of Environmental Selection
  - Select the six extreme solutions
  - Select solution s_4
  - Select solution s_1
  - Select solution s_6



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Experimental Results

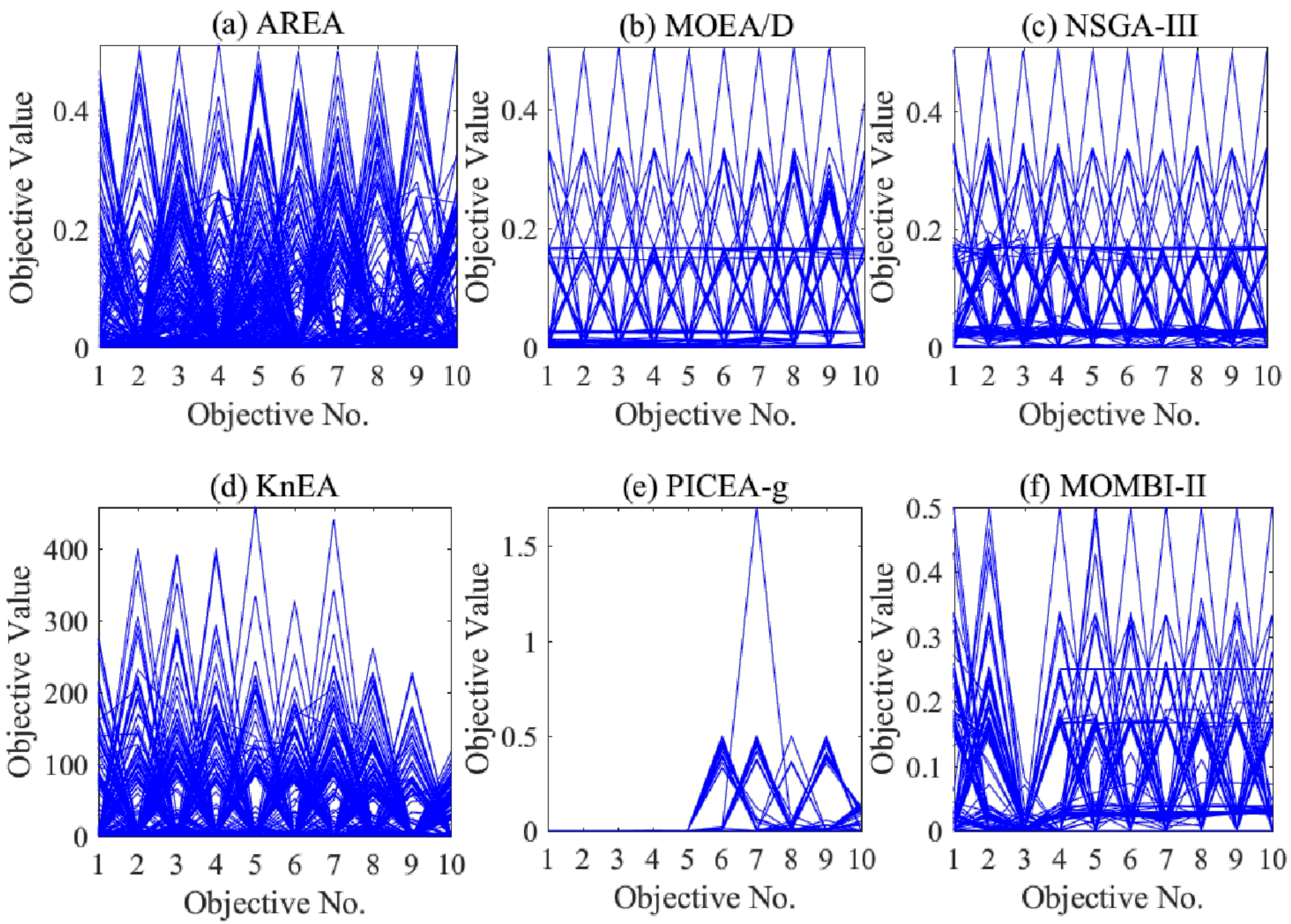
- Results on DTLZ1 to DTLZ7

TABLE IV
THE IGD VALUES OBTAINED BY MOEA/D, NSGA-III, KNEA, PICEA-G, MOMBI-II AND AREA ON DTLZ1 TO DTLZ7. THE BEST RESULT IN EACH ROW IS HIGHLIGHTED.

Problem	Obj.	MOEA/D	NSGA-III	KnEA	PICEA-g	MOMBI-II	AREA
DTLZ1	3	2.85e-2(1.00e-4)−	1.90e-2(1.01e-4)≈	4.61e-2(2.16e-2)−	2.32e-1(1.22e-2)−	1.91e-2(9.06e-5)≈	1.95e-2(8.45e-5)
	5	1.14e-1(3.72e-4)−	6.31e-2(2.66e-4)+	1.79e-1(6.76e-2)−	2.95e-1(5.56e-2)−	6.33e-2(3.64e-4)+	7.55e-2(2.47e-3)
	10	1.85e-1(9.40e-3)−	1.29e-1(2.74e-2)≈	6.66e+0(5.07e+0)−	3.49e-1(5.25e-2)−	2.22e-1(3.63e-2)−	1.24e-1(8.82e-2)
DTLZ2	3	6.85e-2(5.26e-4)−	5.02e-2(2.98e-5)+	6.66e-2(3.42e-3)−	1.00e-1(6.14e-3)−	5.12e-2(5.23e-4)+	5.27e-2(2.85e-4)
	5	3.21e-1(7.00e-4)−	1.95e-1(1.92e-4)+	2.15e-1(5.46e-3)+	2.71e-1(6.43e-3)−	2.00e-1(2.13e-3)+	2.33e-1(5.54e-3)
	10	7.26e-1(5.78e-2)−	4.45e-1(4.61e-2)≈	4.04e-1(4.57e-3)+	4.84e-1(4.27e-2)−	4.23e-1(2.84e-3)+	4.64e-1(7.24e-3)
DTLZ3	3	6.96e-2(3.00e-5)−	5.16e-2(1.45e-3)+	1.01e-1(4.03e-2)−	4.37e-1(3.58e-2)−	5.27e-2(2.63e-3)≈	5.33e-2(3.83e-3)
	5	3.21e-1(6.10e-5)−	2.85e-1(2.54e-1)−	4.55e-1(1.07e-1)−	6.50e-1(4.73e-2)−	1.97e-1(1.51e-3)+	2.34e-1(2.28e-1)
	10	7.03e-1(2.95e-2)−	1.59e+0(2.88e+0)−	2.88e+2(7.84e+1)−	9.99e-1(3.16e-2)−	6.46e-1(1.59e-1)−	6.26e-1(2.90e-1)
DTLZ4	3	4.66e-1(3.55e-1)−	9.94e-2(1.51e-1)−	6.53e-2(2.01e-3)−	2.10e-1(2.36e-1)−	1.26e-1(1.79e-1)−	5.30e-2(4.33e-4)
	5	4.91e-1(1.26e-1)−	2.07e-1(5.03e-2)+	2.15e-1(5.10e-3)+	2.99e-1(6.20e-2)−	2.01e-1(2.04e-3)+	2.41e-1(6.20e-3)
	10	8.17e-1(1.36e-1)−	4.30e-1(3.36e-2)≈	4.10e-1(7.44e-3)+	4.64e-1(5.41e-3)−	4.25e-1(7.22e-4)+	4.90e-1(8.68e-3)
DTLZ5	3	1.23e-2(1.87e-5)−	1.12e-2(1.55e-3)−	9.32e-3(2.56e-3)−	3.04e-2(1.59e-2)−	2.02e-2(3.44e-5)−	5.31e-3(3.73e-4)
	5	4.70e-2(4.00e-4)−	1.49e-1(8.39e-2)−	2.44e-1(1.23e-1)−	2.62e-1(2.71e-1)−	2.72e-1(5.45e-3)−	2.34e-2(1.89e-2)
	10	4.26e-2(4.31e-3)+	4.41e-1(1.18e-1)−	3.76e-1(1.25e-1)−	3.24e-1(5.95e-2)−	5.64e-1(1.69e-1)−	7.24e-2(6.14e-2)
DTLZ6	3	1.24e-2(4.04e-5)−	1.22e-2(1.79e-3)−	5.37e-3(5.70e-4)−	4.73e-2(1.62e-2)−	2.02e-2(1.70e-5)−	4.66e-3(1.13e-4)
	5	4.66e-2(2.70e-3)+	2.29e-1(9.92e-2)−	4.20e-1(1.96e-1)−	1.46e-1(5.25e-2)−	3.19e-1(5.30e-2)−	5.64e-2(6.25e-2)
	10	3.17e-2(3.34e-3)+	3.27e+0(1.33e+0)−	2.43e+0(4.69e-1)−	4.92e-1(7.03e-2)−	5.62e-1(1.68e-1)+	7.49e-2(6.68e-1)
DTLZ7	3	2.29e-1(6.33e-3)−	6.85e-2(2.00e-3)−	8.12e-2(6.44e-2)−	3.32e-1(1.87e-1)−	1.58e-1(1.85e-1)−	6.06e-2(1.30e-3)
	5	6.32e-1(8.37e-2)−	3.22e-1(7.98e-3)≈	2.82e-1(3.03e-2)+	1.37e+0(5.19e-1)−	5.10e-1(2.70e-1)−	3.12e-1(2.93e-2)
	10	1.86e+0(7.06e-1)−	1.59e+0(1.81e-1)≈	8.53e-1(8.96e-3)+	5.21e+0(1.07e-1)−	4.42e+0(9.91e-1)−	1.63e+0(2.10e-1)

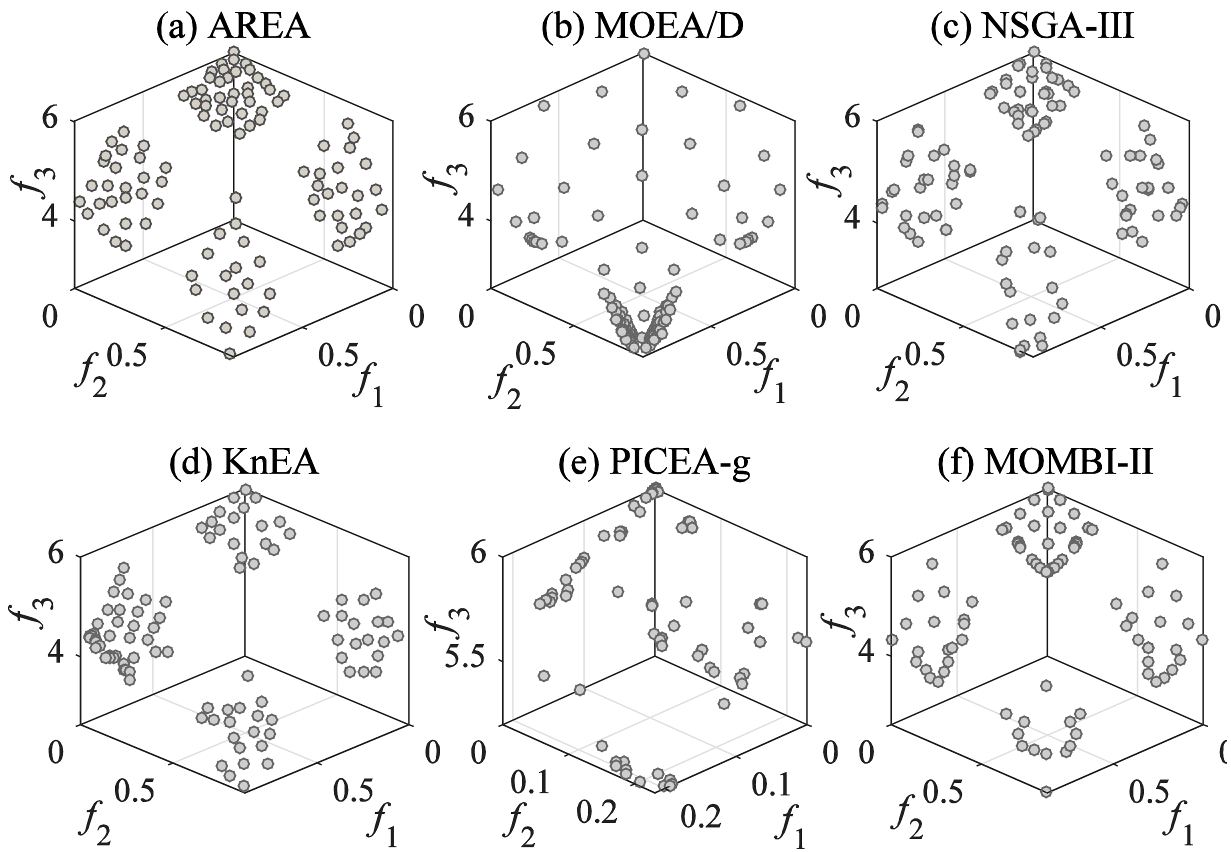
Experimental Results

- Results on DTLZ1



Experimental Results

- Results on DTLZ7



Experimental Results

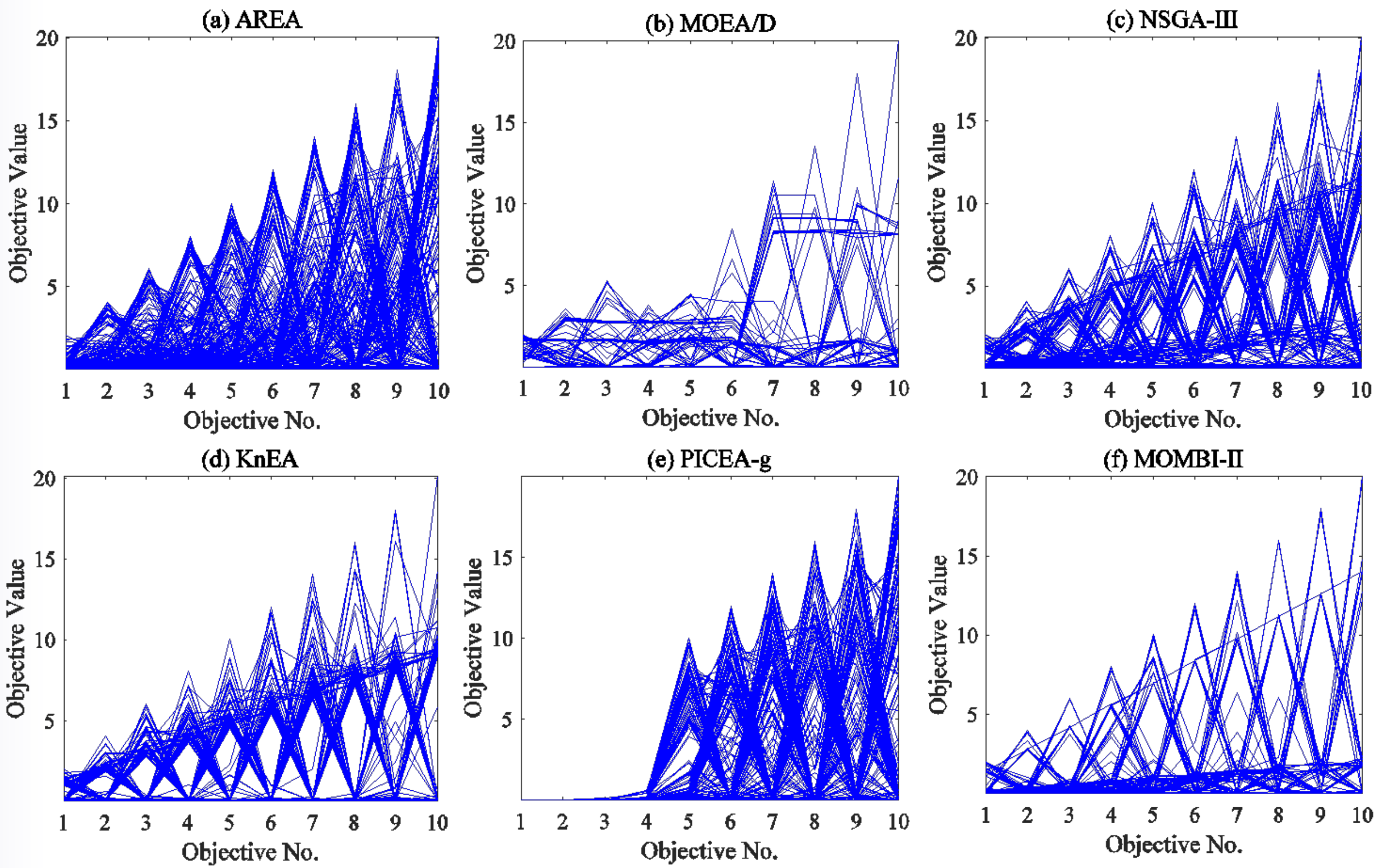
- Results on WFG1 to WFG9

THE HV VALUES OBTAINED BY MOEA/D, NSGA-III, KNEA, PICEA-G, MOMBI-II, AND AREA ON WFG1 TO WFG9. THE BEST RESULT IN EACH ROW IS HIGHLIGHTED.

Problem	Obj.	MOEA/D	NSGA-III	KnEA	PICEA-g	MOMBI-II	AREA
WFG1	3	3.82e+1(1.48e+0)−	4.15e+1(7.71e-1)−	4.35e+1(2.58e-1)−	4.33e+1(6.66e-1)−	4.28e+1(3.15e-1)−	4.41e+1(1.54e-1)
	5	3.49e+3(1.00e+2)−	3.10e+3(1.58e+2)−	3.51e+3(1.19e+2)−	3.82e+3(2.85e+0)≈	3.68e+3(1.94e+2)≈	3.88e+3(1.03e+2)
	10	2.52e+9(4.20e+8)−	3.13e+9(2.27e+8)−	3.64e+9(1.13e+8)−	3.72e+9(7.90e+4)≈	3.71e+9(2.63e+6)≈	3.72e+9(1.66e+6)
WFG2	3	4.27e+1(3.41e-1)−	4.37e+1(5.46e-2)≈	4.37e+1(1.22e-1)≈	4.36e+1(1.24e-1)−	4.17e+1(5.66e-1)−	4.37e+1(6.32e-2)
	5	3.64e+3(3.63e+1)−	3.80e+3(6.88e+0)≈	3.79e+3(6.96e+0)−	3.71e+3(5.93e+1)−	3.81e+3(1.32e+1)+	3.82e+3(5.39e+0)
	10	3.46e+9(2.38e+7)−	3.70e+9(9.58e+6)≈	3.68e+9(3.80e+6)−	3.70e+9(6.55e+6)≈	3.65e+9(5.75e+7)−	3.71e+9(4.65e+6)
WFG3	3	3.07e+0(1.57e-1)−	3.36e+0(6.85e-2)−	3.35e+0(5.65e-2)−	3.72e+0(7.12e-2)+	3.72e+0(2.37e-2)+	3.60e+0(4.61e-2)
	5	1.63e-2(3.57e-2)−	2.60e-1(1.07e-1)−	1.44e-1(7.90e-2)−	1.12e+0(1.18e-1)−	8.99e-4(1.90e-3)−	1.13e+0(1.37e-1)
	10	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	1.43e-5(1.70e-6)+	6.00e-6(2.33e-6)+	2.06e-7(4.72e-7)
WFG4	3	1.74e+1(2.53e-1)−	1.90e+1(9.18e-2)−	1.84e+1(1.28e-1)−	1.87e+1(2.05e-1)−	1.83e+1(2.89e-1)−	1.93e+1(9.94e-2)
	5	1.83e+3(7.63e+1)−	2.36e+3(1.78e+1)+	2.32e+3(2.19e+1)+	2.41e+3(2.63e+1)+	1.70e+3(3.72e+2)−	2.21e+3(4.34e+1)
	10	8.92e+8(1.29e+8)−	3.09e+9(3.21e+7)≈	3.23e+9(1.63e+7)+	2.94e+9(2.17e+8)≈	2.81e+9(2.03e+8)−	2.96e+9(4.09e+7)
WFG5	3	1.68e+1(8.00e-2)−	1.78e+1(1.13e-1)≈	1.69e+1(1.86e-1)−	1.67e+1(1.81e-1)−	1.63e+1(4.19e-1)−	1.76e+1(1.12e-1)
	5	1.68e+3(7.58e+1)−	2.27e+3(7.23e+0)+	2.20e+3(2.64e+1)+	2.18e+3(2.68e+1)+	1.86e+3(1.41e+2)−	2.06e+3(2.61e+1)
	10	9.05e+8(6.87e+7)−	2.96e+9(1.48e+7)+	3.04e+9(1.08e+7)+	2.97e+9(1.07e+8)+	2.19e+9(5.76e+7)−	2.73e+9(3.13e+7)
WFG6	3	1.51e+1(7.61e-1)−	1.66e+1(7.90e-1)−	1.57e+1(5.27e-1)−	1.60e+1(7.17e-1)−	1.59e+1(5.93e-1)−	1.70e+1(6.54e-1)
	5	1.33e+3(1.38e+2)−	2.12e+3(7.17e+1)+	2.02e+3(5.87e+1)≈	2.11e+3(8.66e+1)+	1.56e+3(4.55e+2)−	1.96e+3(6.90e+1)
	10	4.49e+8(1.09e+8)−	2.85e+9(5.67e+7)+	2.92e+9(5.53e+7)+	2.98e+9(6.64e+7)+	2.08e+9(9.78e+7)−	2.66e+9(1.20e+8)
WFG7	3	1.56e+1(1.05e+0)−	1.93e+1(9.11e-2)≈	1.89e+1(2.20e-1)−	1.79e+1(3.84e-1)−	1.82e+1(3.76e-1)−	1.96e+1(4.54e-2)
	5	1.54e+3(9.77e+1)−	2.37e+3(4.57e+1)+	2.43e+3(1.88e+1)+	2.35e+3(2.30e+1)+	2.12e+3(2.64e+2)≈	2.29e+3(3.93e+1)
	10	7.13e+8(9.59e+7)−	3.13e+9(3.61e+7)≈	3.33e+9(1.97e+7)+	3.27e+9(1.18e+8)+	2.69e+9(1.95e+8)−	3.00e+9(6.11e+7)
WFG8	3	1.38e+1(2.99e-1)−	1.49e+1(2.05e-1)≈	1.40e+1(2.22e-1)−	1.25e+1(4.43e-1)−	1.42e+1(1.44e-1)−	1.52e+1(1.45e-1)
	5	1.04e+3(3.26e+2)−	1.84e+3(1.96e+1)+	1.68e+3(3.98e+1)≈	1.75e+3(3.54e+1)≈	1.99e+2(2.95e+1)−	1.74e+3(1.80e+1)
	10	6.46e+7(1.03e+8)−	2.51e+9(7.28e+7)≈	2.40e+9(3.33e+8)−	2.72e+9(6.09e+7)+	1.62e+9(1.10e+8)−	2.50e+9(5.12e+7)
WFG9	3	1.47e+1(1.76e+0)−	1.76e+1(1.15e+0)−	1.81e+1(2.00e-1)≈	1.74e+1(2.28e-1)−	1.74e+1(2.69e-1)−	1.81e+1(1.29e+0)
	5	1.32e+3(2.58e+2)−	2.09e+3(1.19e+2)≈	2.26e+3(2.43e+1)+	2.23e+3(3.10e+1)+	4.47e+2(1.88e+2)−	2.02e+3(1.46e+2)
	10	4.72e+8(2.48e+8)−	2.69e+9(1.73e+8)−	2.98e+9(1.33e+8)+	2.87e+9(9.21e+7)+	2.27e+9(1.01e+8)−	2.70e+9(2.55e+7)

Experimental Results

- Results on WFG4

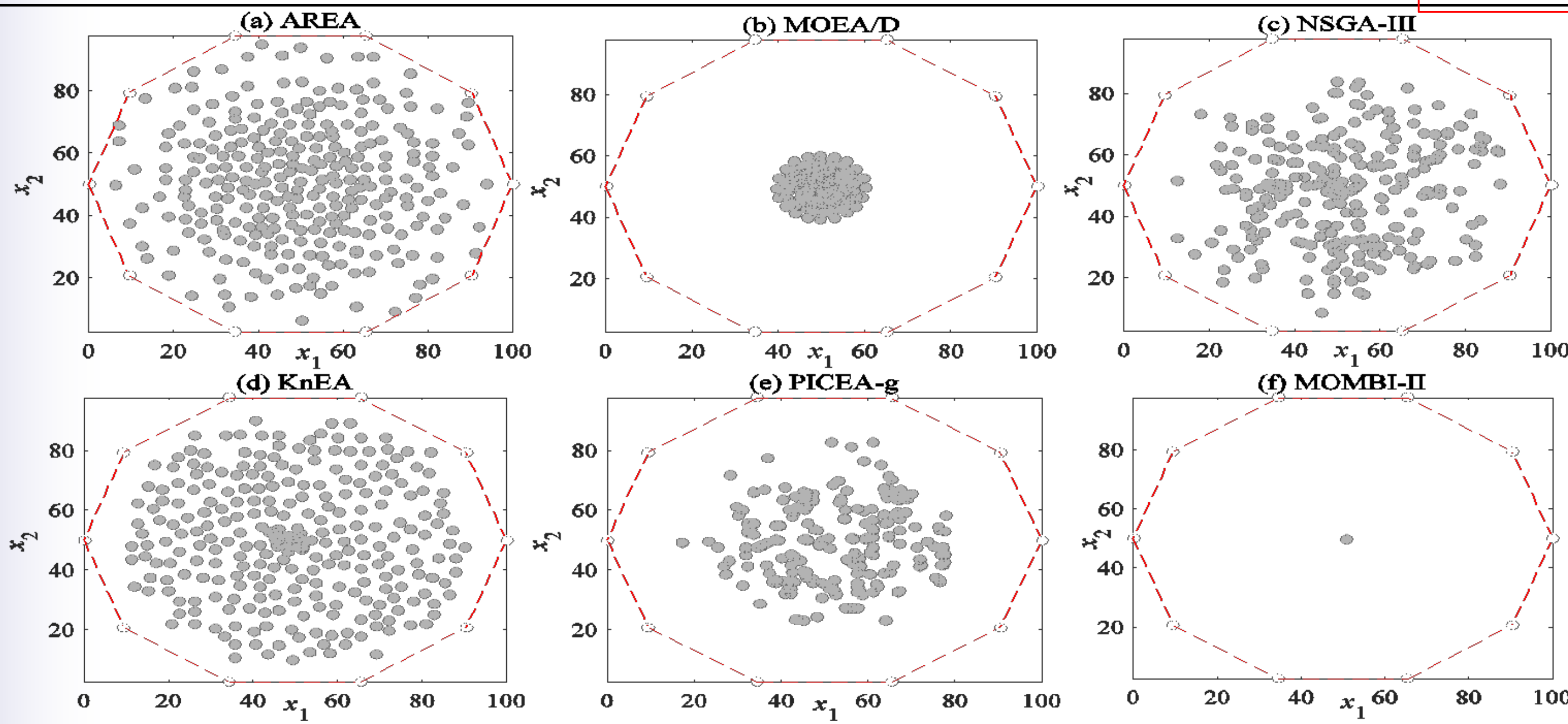


Experimental Results

- Results on ParetoBox Problems

The IGD Values Obtained by MOEA/D, NSGA-III, KnEA, PICEA-g, MOMBI-II and AREA on Pareto-Box Problem. The Best Result in Each Row is Highlighted.

Problem	Obj.	MOEA/D	NSGA-III	KnEA	PICEA-g	MOMBI-II	AREA
ParetoBox	3	3.91e+0(4.85e-3)−	3.30e+0(1.16e-1)−	2.75e+0(8.80e-2)−	3.93e+0(2.20e-1)−	3.93e+0(3.11e-2)−	2.71e+0(4.84e-2)
	5	9.53e+0(5.34e-2)−	5.80e+0(1.61e-1)−	4.19e+0(1.09e-1)−	6.54e+0(3.34e-1)−	1.25e+1(1.61e-1)−	4.10e+0(9.53e-2)
	10	3.80e+1(5.08e-2)−	7.41e+0(8.31e-1)−	4.18e+0(5.21e-2)+	1.09e+1(7.66e-1)−	5.93e+1(1.07e-2)−	4.72e+0(7.99e-2)



Experimental Results

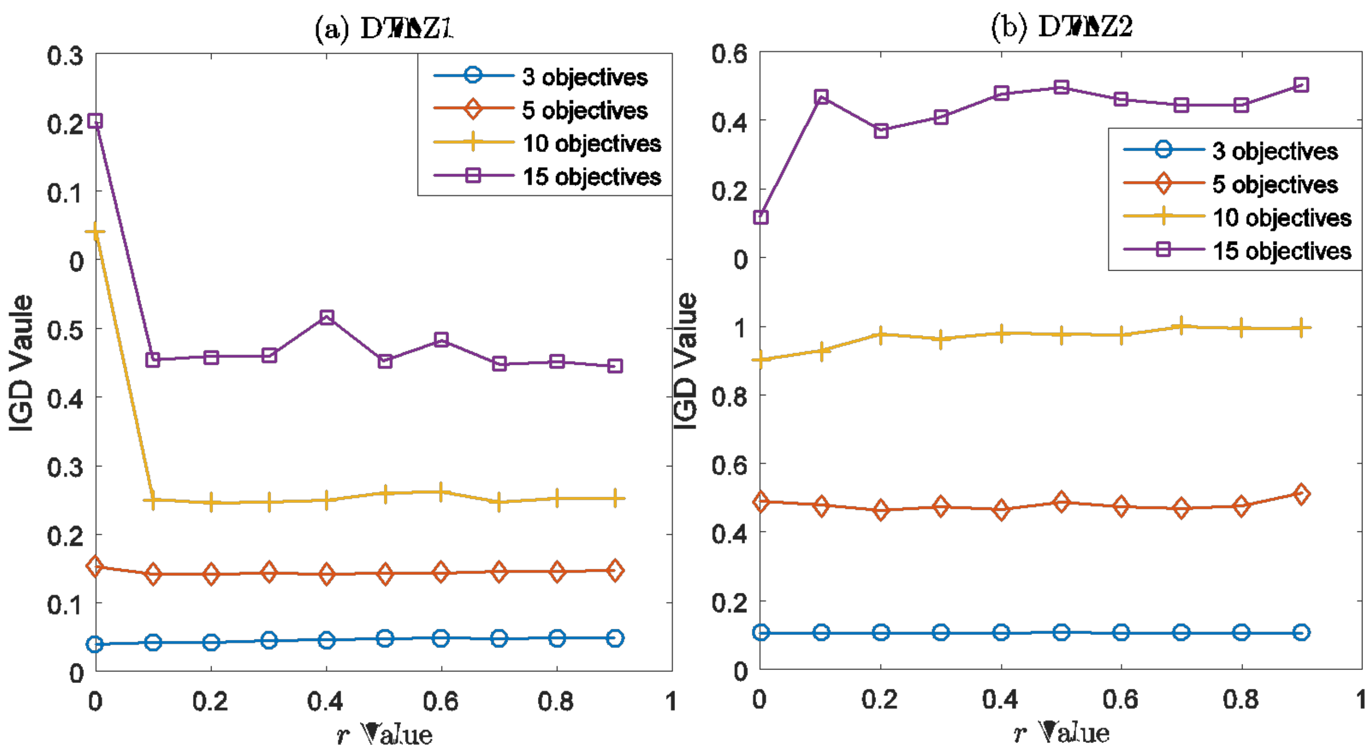
- Impact of the permutation of the projection vectors
  - AREA with fixed projection vectors (AREA)
  - AREA with varying projection vectors (AREA*)
  - AREA with fixed and inverted projection vectors ($AREA^T$)

THE IGD VALUES OBTAINED BY AREA*, $AREA^T$ AND AREA ON 21 TEST INSTANCES. THE BEST RESULT IN EACH ROW IS HIGHLIGHTED.

Problem	Obj.	AREA*	$AREA^T$	AREA
DTLZ1	3	2.01e-2(1.78e-4)−	2.00e-2(7.85e-5)≈	2.00e-2(8.45e-5)
	5	8.86e-2(4.70e-2)−	8.50e-2(3.86e-2)≈	7.55e-2(2.47e-3)
	10	3.41e-1(1.77e-1)≈	4.08e-1(2.23e-1)≈	3.61e-1(1.66e-1)
DTLZ2	3	5.33e-2(3.47e-4)−	5.28e-2(3.28e-4)≈	5.27e-2(2.85e-4)
	5	2.38e-1(5.54e-3)−	2.32e-1(5.16e-3)≈	2.33e-1(5.54e-3)
	10	4.55e-1(5.78e-3)+	4.67e-1(8.52e-3)≈	4.64e-1(7.24e-3)
DTLZ5	3	6.37e-3(6.37e-4)−	5.42e-3(5.18e-4)≈	5.31e-3(3.73e-4)
	5	8.91e-2(3.29e-2)+	1.47e-1(3.13e-2)≈	1.46e-1(3.11e-2)
	10	2.61e-1(9.73e-2)+	4.06e-1(8.83e-2)≈	4.06e-1(9.34e-2)
DTLZ7	3	7.77e-2(6.40e-2)+	7.54e-2(6.45e-2)≈	1.05e-1(1.07e-1)
	5	3.63e-1(4.69e-2)−	3.29e-1(5.11e-2)≈	3.30e-1(3.80e-2)
	10	1.90e+0(2.87e-1)−	1.70e+0(2.38e-1)≈	1.74e+0(1.82e-1)
WFG1	3	1.80e-1(2.58e-2)−	1.60e-1(1.18e-2)−	1.54e-1(1.49e-2)
	5	9.69e-1(1.47e-1)−	7.33e-1(7.02e-2)≈	7.30e-1(5.81e-2)
	10	1.57e+0(1.12e-1)−	1.44e+0(6.86e-2)≈	1.44e+0(8.28e-2)
WFG3	3	1.09e-1(1.17e-2)−	9.69e-2(7.04e-3)≈	9.80e-2(9.75e-3)
	5	4.48e-1(4.69e-2)−	3.21e-1(2.89e-2)≈	3.21e-1(3.08e-2)
	10	4.62e-1(1.65e-1)≈	4.21e-1(4.67e-2)≈	4.09e-1(4.20e-2)
WFG4	3	2.16e-1(2.24e-3)≈	2.15e-1(2.58e-3)≈	2.15e-1(2.24e-3)
	5	1.29e+0(4.36e-2)≈	1.29e+0(3.11e-2)≈	1.28e+0(1.86e-2)
	10	4.34e+0(8.92e-2)≈	4.34e+0(7.35e-2)≈	4.36e+0(8.94e-2)

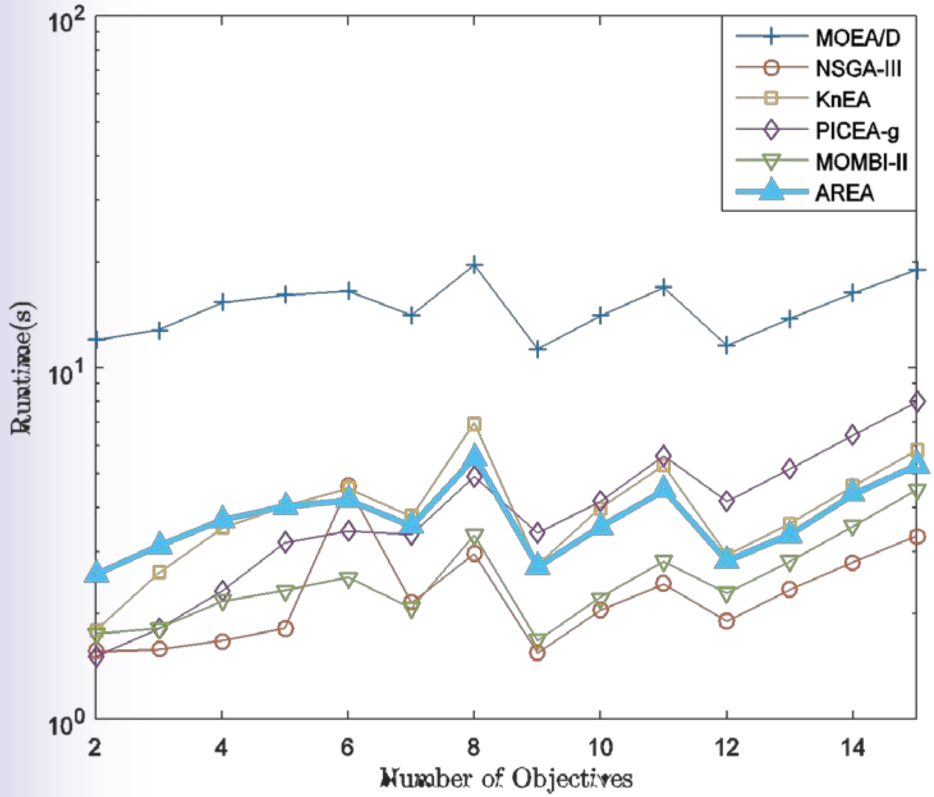
Experimental Results

- Sensitivity of the parameter *r*



Experimental Results

- Runtime Performance
  - Computational Complexity ($O(GMN^2)$)



Obj.	2	3	4	5	6	7	8
p_1	99	13	7	5	4	3	3
p_2	0	0	0	0	1	2	2
N	10	105	120	126	132	112	156
Obj.	9	10	11	12	13	14	15
p_1	2	2	2	2	2	2	2
p_2	2	2	2	1	1	1	1
N	90	110	132	90	104	119	135

Settings of population size for the six compared algorithms

Outline

- Many-Objective Optimization
- Radial Projection
- The Proposed AREA
- Experimental Results
- Conclusion

Conclusion

- Diversity-first-convergence-second mechanism can be used for solving MaOPs
- The proposed AREA is competitive with other MaOEAs on many-objective optimization
- The proposed AREA is efficiency

Thank you