

## **Evolutionary Multiobjective Optimization Driven by Generative Adversarial Networks (GANs)**

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#### **Research Interests – Evolving machine intelligence (EMI)**





- Background
- Motivation
- Proposed Algorithm
- Experimental Results
- Conclusion and Future Work





### • Background

- Multiobjective optimization
- Generative adversarial networks (GANs)
- Model based evolutionary algorithms (MBEAs)
- Motivation
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#### Multiobjective Optimization





The mathematical formulation of a multiobjective optimization problem (MOP):

 $\min_{\mathbf{x}} f_i(\mathbf{x}) \quad i = 1, 2, \dots, M$ s.t.  $\mathbf{x} \in [\mathbf{a}, \mathbf{b}],$ 

where  $\mathbf{x} = (x_1, x_2, ..., x_D)$  is the **decision vector** which consists a number of *D* decision variables,  $f_i(\mathbf{x})$  are the **optimization objectives**, **a** and **b** are the **box constraints**.

- **x** is **Pareto optimal** *iff* no other solution Pareto dominates it.
- Pareto set (**PS**): the set of all Pareto optimal solutions in the decision space.
- Pareto front (**PF**): the image of the PS in the objective space.
- Regularity: under certain smoothness assumptions, the PS of a continuous MOP is a piecewise continuous manifold.



GANs are generative models that produce a model distribution  $P_{\hat{x}}$  (i.e., the distribution of the fake/generated data) that mimics a target distribution  $P_x$ (i.e., the distribution of the real/given data).



## ■ Adversarial training: D seeks to distinguish the real and generated samples, while G aims to fool D.



Heatmaps of generated distributions in different stage of the training.

## Algorithm 1 Training of the GANs

#### Input:

4:

 $P_{\mathbf{x}}$  (given data),  $P_{\mathbf{z}}$  (Gaussian noise), m (batch size).

- 1: for total number of training iterations do
- 2:  $\mathbf{X}' \leftarrow P_{\mathbf{x}}$

3: for 
$$i \leftarrow 1 : |P_{\mathbf{x}}|/m$$
 do

/\*\*\*\*\* Update the discriminator \*\*\*\*/

- 5:  $\mathbf{T} \leftarrow \text{Randomly sample } m \text{ data points from } \mathbf{X}'$
- 6:  $\mathbf{X'} \leftarrow \mathbf{X'} ackslash \mathbf{T}$ 
  - $\mathbf{Z} \leftarrow \text{Sample } m$  noise data points from  $P_{\mathbf{z}}$
  - Update the discriminator according to (4) by using  $\mathbf{T}$  and  $\mathbf{Z}$
  - /\*\*\*\*\*\* Update the generator \*\*\*\*\*\*/
  - $\mathbf{Z} \leftarrow \mathbf{Sample} \ m$  noise data points from  $P_{\mathbf{z}}$
  - Update the generator according to (5) by using **Z** end



MBEAs aims to equip the EAs with *learning abilities*, which can be roughly classified into three categories.

#### **Estimation of Distribution:**

Estimate the distribution of the promising candidate solutions by training and sampling models in the decision space.

#### **Inverse Modeling:**

sample points in the decision space and then build inverse models to map them back to the decision space.

#### **Surrogate Modeling:**

 computationally efficient models are introduced for replacing the computationally expensive models.



Models can be used in EAs for different purposes

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Motivation



Machine learning models are useful but expensive to drive evolutionary algorithms: (1) Require enough data samples, (2) Handle problems with small scale of decision variables or objectives. Offspring generation is as important as selection.

#### **GANs are suitable for EAs:**

- The PS of an MOP is naturally a regular distribution in the decision space;
- The samples can be divided into fake and real ones, which is somehow consistent with the good and bad candidate solutions in EA.
- GANs can used to sampling and distinguishing promising candidate solutions.
- GANs are able to learn high-dimensional distributions efficiently with limited training data.

#### **D** Previous work:

- Classification based MBEA
- Balance between convergence and diversity



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#### General framework of GMOEA



#### General framework

- Solution Classification
  - ✓ Provide "*true*" and "*fake*" datasets
  - $\checkmark$  Provide selection pressure
- Model Training
  - $\checkmark$  Reuse the better solutions
  - ✓ Modify the noise
- Offspring Generation
  - ✓ **Sample** solutions from the GANs
  - ✓ Hybrid crossover and sampling
- > EA Framework
  - ✓ SPEA2 for diversity maintenance

#### Algorithm 2 General Framework of GMOEA

#### Input:

- N (population size), m (batch size)
- 1:  $P \leftarrow$  Initialize a population of size N
- 2:  $GAN \leftarrow$  Initialize the GANs
- 3: while termination criterion not fulfilled do
- 4:  $\mathbf{X} \leftarrow$  Solution Classification /\*Half of the solutions in *P* are classified as *fake* samples\*/
- 5:  $\mathbf{net} \leftarrow \mathbf{Model \ Training \ /*Use \ X}$  to train the model\*/
- 6:  $Q \leftarrow \text{Offspring Reproduction /*Generate } N \text{ offspring solutions by the proposed reproduction method*/}$
- 7:  $P \leftarrow$  Environmental Selection /\*Select N solutions from the combination of P and Q\*/
- 8: **end**
- 9: **Return:** *P*

#### Model training in GMOEA



#### □ Main idea

- ✓ Learn the distribution of the good solutions ("*real*" ones)
- ✓ Push away from the bad ones ("*fake*" ones)
- Prefer model with higher accuracy in distinguishing the *fake* samples

Model Training

- ✓ Obtain information from the "*real*" ones
- $\checkmark$  Use multivariate normal distribution
- $\checkmark$  Train the discriminator

 $\max_{D} V(D) = \mathsf{E}_{\mathbf{r} \in P_{\mathbf{r}}}[log(D(\mathbf{r}))] + \mathsf{E}_{\mathbf{f} \in P_{\mathbf{f}}}[log(1 - D(\mathbf{f}))] + \mathsf{E}_{\mathbf{z} \in P_{\mathbf{z}}}[log(1 - D(G(\mathbf{z})))]$ 

 $\checkmark$  Train the generator

$$\nabla \theta_g \frac{1}{m} \sum_{i=1}^m log \left( 1 - D(G(\mathbf{z}_i)) \right)$$



The general scheme of model training in the proposed method.

#### Algorithm 4 Model Training Input: X (given data), m (number of samplings).

1:  $\mu \leftarrow mean(P_r)$  /\*Mean vector of the data\*/ 2:  $\Sigma \leftarrow cov(P_r)$  /\*Covariance matrix of the data\*/ 3: for total number of training iterations do 4: Randomly sample *m* samples { $x_1, ..., x_m$ } from X 5: { $z_1, ..., z_m$ }  $\leftarrow multivariate\_normal(m, \mu, \Sigma)$ 6: Update the discriminator according to (10) 7: { $z_1, ..., z_m$ }  $\leftarrow multivariate\_normal(m, \mu, \Sigma)$ 8: Update the generator according to (4) 9: end

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  - Effectiveness of the training method
  - General performance
  - Ablation study
- Conclusion and Future Work



#### Effectiveness of the training method



#### **Training loss trajectories**

- > More stable
- Better generation capability
- □ Offspring generation at different generations
  - Better distributed
  - Better converged

icinial GAN

200 200

F of IMF4



Modified GAI

PE of IMEA

inginal GAN

400 150



The trajectories of generator and discriminator's training losses of the original and our modified GAN during the evolution, respectively.



The offsprings generated by the original GANs and our modified GANs at different iterations of the evolution.

400 150

Original GAN:

PF of IMF4



#### Experimental results on IMF problems with 30, 50, and 200 decision variables (FEs: {5000, 15000, 30000})

THE IGD RESULTS OBTAINED BY NSGA-II, MOEA/D-DE, MOEA/D-CMA, IM-MOEA, GDE3, SPEA2, AND GMOEA ON 40 IMF TEST INSTANCES. THE BEST RESULT IN EACH ROW IS HIGHLIGHTED. THE HV RESULTS OBTAINED BY NSGA-II, MOEA/D-DE, MOEA/D-CMA, IM-MOEA, GDE3, SPEA2, AND GMOEA ON 40 IMF TEST INSTANCES. THE BEST RESULT IN EACH ROW IS HIGHLIGHTED.

Problem	Dim	NSGA-II	MOEA/D-DE	MOEA/D-CMA	IM-MOEA	GDE3	SPEA2	GMOEA	Problem	Dim	NSGA-II	MOEA/D-DE	MOEA/D-CMA	IM-MOEA	GDE3	SPEA2	GMOEA
IMF1	30	2.75e-1(3.56e-2)+	5.12e-1(8.51e-2)-	2.92e-1(4.07e-2)+	1.17e-1(2.86e-2)+	9.92e-1(2.87e-1)-	2.89e-1(4.73e-2)+	4.46e-1(3.86e-2)		30	5.43e-1(3.16e-2)+	1.85e-1(6.65e-2)-	4.00e-1(4.98e-2)-	7.18e-1(1.99e-2) +	2.45e-2(2.87e-2)-	5.19e-1(4.64e-2)≈	5.08e-1(2.55e-2)
	50	3.13e-1(3.67e-2)+	5.43e-1(8.84e-2)-	2.26e-1(2.74e-2)+	1.24e-1(3.46e-2)+	1.10e+0(2.43e-1)-	3.25e-1(3.52e-2)+	4.67e-1(4.44e-2)	-1(4.44e-2) IMF1 -1(5.10e-2)	50	5.50e-1(3.43e-2)+	1.69e-1(6.34e-2)-	4.99e-1(4.09e-2)≈	7.29e-1(1.90e-2)+	1.50e-2(4.01e-2)-	5.45e-1(1.93e-2)+	4.87e-1(3.26e-2)
	100	3.53e-1(3.20e-2)+	1.06e+0(1.62e-1)-	3.76e-1(4.08e-2)+	2.29e-1(3.52e-2)+	2.08e+0(3.01e-1)-	3.85e-1(3.25e-2)+	4.87e-1(5.10e-2)		100	4.80e-1(3.68e-2)≈	6.00e-3(1.76e-2)-	3.11e-1(3.75e-2)-	6.29e-1(2.16e-2)+	0.00e+0(0.00e+0)-	4.46e-1(4.22e-2)≈	4.65e-1(3.50e-2)
	200	3.85e-1(2.40e-2)+	1.29e+0(1.42e-1)-	4.06e-1(3.43e-2)+	2.61e-1(3.82e-2)+	2.57e+0(2.23e-1)-	4.31e-1(2.63e-2)+	5.44e-1(5.43e-2)	200	4.33e-1(2.75e-2)≈	0.00e+0(0.00e+0)-	2.7/e-1(2.98e-2)-	6.06e-1(2.39e-2)+	0.00e+0(0.00e+0)-	3.80e-1(3.59e-2)-	4.0/e-1(8.18e-2)	
IMF2	30	4.69e-1(5.60e-2)+	7.50e-1(1.67e-1)-	4.52e-1(7.56e-2)+	2.15e-1(7.97e-2)+	2.01e+0(6.60e-1)-	4.72e-1(4.76e-2)+	6.10e-1(1.14e-6)	-	30	4.65e-2(3.13e-2)-	1.70e-2(3.69e-2)-	1.00e-1(4.24e-2)≈	2.71e-1(5.94e-2)+	0.00e+0(0.00e+0)-	4.05e-2(3.50e-2)-	1.10e-1(2.85e-7)
	50	4.78e-1(2.95e-2)+	7.17e-1(1.66e-1)-	3.28e-1(3.44e-2)+	2.84e-1(9.53e-2)+	1.92e+0(4.60e-1)-	4.78e-1(2.97e-2)+	6.10e-1(1.14e-6)	IMF2	50	5.90e-2(2.45e-2)-	2.15e-2(2.85e-2)-	1.92e-1(2.88e-2)+	2.32e-1(5.86e-2)+	0.00e+0(0.00e+0)-	5.35e-2(2.64e-2)-	1.10e-1(2.85e-7)
	100	5.29e-1(3.08e-2)+	1.66e+0(3.47e-1)-	5.29e-1(5.96e-2)+	3.96e-1(5.81e-2)+	3.42e+0(4.70e-1)-	5.67e-1(3.80e-2)+	6.10e-1(1.14e-6)		100	6.50e-3(7.45e-3)-	0.00e+0(0.00e+0)-	6.70e-2(2.75e-2)-	1.41e-1(2.92e-2)+	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	1.10e-1(2.85e-7)
	200	5.75e-1(3.95e-2)+	2.28e+0(2.47e-1)-	5.93e-1(6.10e-2)≈	4.11e-1(3.16e-2)+	4.28e+0(3.25e-1)-	6.48e-1(5.17e-2)+	6.81e-1(2.29e-1)		200	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	4.35e-2(2.30e-2)-	1.36e-1(1.39e-2)+	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	9.35e-2(4.03e-2)
IMF3	30	3.02e-1(9.01e-2)-	6.73e-1(2.58e-1)-	4.61e-1(5.61e-2)-	1.58e-1(2.97e-2)-	2.26e+0(5.06e-1)-	3.25e-1(9.57e-2)-	1.00e-2(1.78e-8)	IMF3	30	1.32e-1(5.32e-2)-	2.85e-2(6.27e-2)-	4.75e-2(1.92e-2)-	2.39e-1(2.88e-2)-	0.00e+0(0.00e+0)-	1.14e-1(5.15e-2)-	4.24e-1(5.03e-3)
	50	1.74e-1(4.43e-2)-	7.59e-1(2.02e-1)-	3.89e-1(2.07e-2)-	1.29e-1(3.33e-2)-	2.96e+0(5.62e-1)-	2.12e-1(5.22e-2)-	1.00e-2(1.78e-8)		50	2.34e-1(4.20e-2)-	7.50e-3(1.12e-2)-	7.60e-2(1.14e-2)-	2.69e-1(3.20e-2)-	0.00e+0(0.00e+0)-	1.99e-1(4.23e-2)-	4.28e-1(4.10e-3)
	100	3.26e-1(6.12e-2)-	2.03e+0(2.83e-1)-	5.41e-1(4.80e-2)-	2.42e-1(4.59e-2)-	5.78e+0(8.25e-1)-	3.57e-1(8.06e-2)-	1.00e-2(1.78e-8)		100	1.19e-1(3.75e-2)-	0.00e+0(0.00e+0)-	2.50e-2(1.10e-2)-	1.71e-1(3.43e-2)-	0.00e+0(0.00e+0)-	1.01e-1(4.12e-2)-	4.24e-1(5.03e-3)
	200	3.76e-1(6.04e-2)-	2.74e+0(2.63e-1)-	5.72e-1(5.99e-2)-	2.37e-1(2.83e-2)-	7.25e+0(6.55e-1)-	4.11e-1(5.90e-2)-	7.60e-2(1.60e-1)		200	9.05e-2(3.03e-2)-	0.00e+0(0.00e+0)-	1.95e-2(1.23e-2)-	1.73e-1(2.15e-2)-	0.00e+0(0.00e+0)-	7.45e-2(2.72e-2)-	3.64e-1(1.26e-1)
IMF4	30	1.17e+0(3.48e-1)-	2.75e+0(8.05e-1)-	1.43e+0(2.53e-1)-	2.18e+0(4.68e-1)-	7.19e+0(2.53e+0)-	1.21e+0(3.11e-1)-	5.21e-1(1.23e-2)		30	5.00e-4(2.24e-3)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	3.50e-3(1.18e-2)-	4.35e-1(2.26e-2)
	50	1.54e+0(4.63e-1)-	3.78e+0(1.08e+0)-	1.44e+0(2.36e-1)-	2.96e+0(4.95e-1)-	1.85e+1(5.25e+0)-	1.47e+0(4.31e-1)-	5.37e-1(5.71e-3)	IME4	50	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	4.54e-1(9.95e-3)
	100	6.62e+0(1.56e+0)-	1.91e+1(3.76e+0)-	4.71e+0(8.46e-1)-	1.33e+1(2.19e+0)-	8.23e+1(1.59e+1)-	5.84e+0(9.71e-1)-	6.57e-1(5.23e-1)	1.011 4	100	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	4.35e-1(1.02e-1)
	200	1.95e+1(2.25e+0)-	4.65e+1(5.99e+0)-	9.62e+0(1.79e+0)-	3.21e+1(5.33e+0)-	2.14e+2(1.82e+1)-	1.52e+1(1.88e+0)-	1.01e+0(2.08e+0)		200	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	4.35e-1(1.02e-1)
IMF5	30	9.90e-2(1.02e-2)-	1.39e-1(8.13e-3)-	1.40e-1(1.32e-2)-	7.55e-2(8.87e-3)≈	1.10e-1(2.21e-2)-	9.70e-2(1.08e-2)-	7.55e-2(1.10e-2)		30	7.03e-1(1.39e-2)-	6.13e-1(1.14e-2)-	6.09e-1(1.85e-2)-	7.23e-1(1.42e-2)-	6.66e-1(4.81e-2)-	7.04e-1(1.23e-2)-	7.58e-1(1.69e-2)
	50	1.08e-1(1.28e-2)-	1.35e-1(1.15e-2)-	1.33e-1(1.42e-2)-	6.80e-2(6.16e-3)+	1.29e-1(1.17e-2)-	1.09e-1(1.04e-2)-	8.15e-2(1.35e-2)	IMES	50	7.07e-1(1.21e-2)-	6.23e-1(2.08e-2)-	6.23e-1(2.30e-2)-	7.40e-1(9.99e-3)-	6.28e-1(1.84e-2)-	7.06e-1(1.15e-2)-	7.53e-1(1.89e-2)
	100	1.37e-1(8.75e-3)≈	1.68e-1(7.68e-3)-	1.62e-1(8.13e-3)-	1.02e-1(6.16e-3)≈	1.68e-1(8.94e-3)-	1.43e-1(7.33e-3)-	1.20e-1(3.20e-2)	INIT 5	100	6.60e-1(1.05e-2)-	5.70e-1(1.03e-2)-	5.83e-1(9.10e-3)-	6.83e-1(1.17e-2)-	5.74e-1(1.23e-2)-	6.54e-1(9.88e-3)-	7.10e-1(3.50e-2)
	200	1.60e-1(9.45e-3)-	1.85e-1(5.13e-3)-	1.66e-1(9.99e-3)-	1.13e-1(7.33e-3)≈	1.88e-1(6.96e-3)-	1.75e-1(1.73e-2)-	1.11e-1(1.80e-2)		200	6.32e-1(1.01e-2)-	5.46e-1(5.98e-3)-	5.78e-1(1.21e-2)-	6.68e-1(8.13e-3)-	5.46e-1(9.40e-3)-	6.14e-1(1.43e-2)-	7.13e-1(3.28e-2)
IMF6	30	1.77e-1(2.32e-2)-	1.93e-1(1.87e-2)-	1.92e-1(1.39e-2)-	1.01e-1(1.17e-2)+	1.61e-1(4.18e-2)-	1.80e-1(1.84e-2)-	1.17e-1(1.53e-2)	IME6	30	2.98e-1(2.31e-2)-	3.39e-1(2.07e-2)-	3.39e-1(1.70e-2)-	4.04e-1(1.35e-2)≈	3.72e-1(4.76e-2)-	2.95e-1(1.76e-2)-	4.08e-1(1.48e-2)
	50	1.92e-1(2.19e-2)-	1.94e-1(2.50e-2)-	1.80e-1(2.78e-2)-	9.70e-2(8.01e-3)+	1.96e-1(2.21e-2)-	2.02e-1(1.93e-2)-	1.25e-1(1.15e-2)		50	2.80e-1(2.24e-2)-	3.35e-1(2.52e-2)-	3.50e-1(2.64e-2)-	4.03e-1(9.67e-3)≈	3.34e-1(2.66e-2)-	2.70e-1(2.08e-2)-	4.08e-1(1.37e-2)
	100	2.70e-1(2.83e-2)-	2.37e-1(8.01e-3)-	2.23e-1(1.49e-2)-	1.41e-1(6.86e-3)≈	2.59e-1(1.36e-2)-	2.79e-1(2.52e-2)-	1.77e-1(7.11e-2)	IMPO	100	1.97e-1(2.80e-2)-	2.95e-1(1.10e-2)-	3.08e-1(1.61e-2)≈	3.60e-1(9.45e-3)+	2.67e-1(1.53e-2)-	1.86e-1(2.35e-2)-	3.16e-1(7.80e-2)
	200	3.19e-1(2.48e-2)-	2.59e-1(5.53e-3)-	2.45e-1(1.23e-2)-	1.54e-1(6.81e-3)+	2.80e-1(7.95e-3)-	3.32e-1(3.62e-2)-	1.90e-1(3.50e-2)		200	1.57e-1(1.89e-2)-	2.73e-1(6.57e-3)-	2.83e-1(1.49e-2)≈	3.47e-1(8.01e-3)+	2.46e-1(8.21e-3)-	1.42e-1(3.27e-2)-	2.97e-1(3.92e-2)
IMF7	30	1.79e-1(1.79e-2)-	2.83e-1(1.16e-2)-	2.87e-1(5.87e-3)-	2.45e-1(7.61e-3)-	3.00e-1(1.03e-2)-	1.98e-1(2.59e-2)-	6.40e-2(3.03e-2)		30	2.35e-1(1.54e-2)-	1.62e-1(8.34e-3)-	1.58e-1(5.23e-3)-	1.91e-1(6.41e-3)-	1.38e-1(8.51e-3)-	2.19e-1(2.16e-2)-	3.46e-1(4.31e-2)
	50	1.58e-1(1.94e-2)-	2.83e-1(6.57e-3)-	2.84e-1(5.03e-3)-	2.32e-1(1.28e-2)-	2.94e-1(8.26e-3)-	1.64e-1(2.09e-2)-	5.10e-2(5.48e-2)	IMF7	50	2.54e-1(2.06e-2)-	1.61e-1(2.24e-3)-	1.61e-1(5.10e-3)-	2.03e-1(1.02e-2)-	1.45e-1(6.07e-3)-	2.49e-1(2.34e-2)-	3.69e-1(6.40e-2)
	100	2.03e-1(2.20e-2)-	2.91e-1(2.24e-3)-	2.93e-1(4.89e-3)-	2.50e-1(6.49e-3)-	3.05e-1(6.07e-3)-	2.09e-1(1.59e-2)-	1.65e-2(1.46e-2)		100	2.10e-1(2.22e-2)-	1.51e-1(3.08e-3)-	1.50e-1(6.49e-3)-	1.84e-1(6.81e-3)-	1.37e-1(4.70e-3)-	2.06e-1(1.57e-2)-	4.11e-1(2.24e-2)
	200	2.39e-1(2.02e-2)-	2.94e-1(5.10e-3)-	2.95e-1(5.13e-3)-	2.53e-1(8.65e-3)-	3.08e-1(5.23e-3)-	2.42e-1(1.98e-2)-	7.25e-2(8.58e-2)		200	1.78e-1(1.74e-2)-	1.49e-1(3.08e-3)-	1.48e-1(5.23e-3)-	1.80e-1(9.45e-3)-	1.40e-1(5.70e-17)-	1.77e-1(1.49e-2)-	3.46e-1(9.47e-2)
IMF8	30	7.37e-1(1.18e-1)-	6.44e-1(3.60e-2)-	6.12e-1(1.05e-1)-	5.59e-1(4.83e-2)-	6.92e-1(1.81e-1)-	7.44e-1(1.24e-1)-	3.41e-1(1.90e-2)		30	3.50e-3(8.13e-3)-	1.43e-1(2.07e-2)-	1.71e-1(1.10e-1)-	8.50e-2(3.07e-2)-	1.53e-1(1.66e-1)-	3.50e-3(8.13e-3)-	3.55e-1(6.92e-2)
	50	9.80e-1(1.20e-1)-	6.71e-1(2.85e-2)-	6.81e-1(4.24e-2)-	6.55e-1(4.56e-2)-	9.33e-1(7.64e-2)-	1.00e+0(1.48e-1)-	3.58e-1(1.15e-2)	IMES	50	0.00e+0(0.00e+0)-	1.28e-1(1.92e-2)-	1.20e-1(1.38e-2)-	2.00e-2(2.10e-2)-	2.50e-3(1.12e-2)-	0.00e+0(0.00e+0)-	4.66e-1(4.80e-2)
	100	1.74e+0(1.62e-1)-	7.35e-1(5.38e-2)-	7.74e-1(3.13e-2)-	1.28e+0(7.10e-2)-	1.72e+0(2.64e-1)-	2.43e+0(2.16e-1)-	4.85e-1(8.46e-2)	імгә	100	0.00e+0(0.00e+0)-	5.15e-2(4.37e-2)-	1.40e-2(1.23e-2)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	3.40e-1(1.21e-1)
	200	4.00e+0(6.32e-1)-	8.55e-1(1.10e-1)≈	8.88e-1(3.26e-2)≈	2.28e+0(2.16e-1)-	3.40e+0(4.49e-1)-	5.96e+0(4.13e-1)-	1.31e+0(1.55e+0)		200	0.00e+0(0.00e+0)-	1.05e-2(1.85e-2)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	0.00e+0(0.00e+0)-	2.24e-1(2.20e-1)
IMF9	30	1.10e-1(1.49e-2)-	2.91e-1(5.01e-2)-	3.28e-1(5.37e-2)-	2.09e-1(2.20e-2)-	2.50e-1(3.63e-2)-	1.17e-1(1.38e-2)-	7.30e-2(2.64e-2)		30	6.77e-1(2.08e-2)-	4.43e-1(5.79e-2)-	3.92e-1(6.29e-2)-	5.32e-1(3.54e-2)-	4.86e-1(4.84e-2)-	6.63e-1(2.03e-2)-	7.78e-1(3.24e-2)
	50	1.07e-1(1.92e-2)-	2.91e-1(4.21e-2)-	3.70e-1(4.42e-2)-	1.78e-1(2.61e-2)-	2.87e-1(4.50e-2)-	1.10e-1(1.08e-2)-	8.75e-2(2.69e-2)	D.IEO	50	6.87e-1(3.10e-2)-	4.41e-1(4.96e-2)-	3.46e-1(4.49e-2)-	5.82e-1(3.93e-2)-	4.38e-1(5.59e-2)-	6.81e-1(1.81e-2)-	7.60e-1(2.75e-2)
	100	1.46e-1(9.40e-3)-	4.44e-1(4.83e-2)-	4.80e-1(3.34e-2)-	2.89e-1(2.61e-2)-	3.80e-1(4.26e-2)-	1.48e-1(8.75e-3)-	1.16e-1(3.27e-2)	IMF9	100	6.28e-1(1.28e-2)-	2.78e-1(4.39e-2)-	2.41e-1(2.70e-2)-	4.25e-1(3.17e-2)-	3.32e-1(4.29e-2)-	6.23e-1(1.26e-2)-	7.37e-1(2.23e-2)
	200	1.73e-1(8.01e-3)-	5.50e-1(2.03e-2)-	5.26e-1(3.35e-2)-	2.95e-1(2.91e-2)-	4.96e-1(2.54e-2)-	1.71e-1(1.04e-2)-	1.41e-1(5.95e-2)		200	5.89e-1(1.12e-2)-	1.88e-1(1.54e-2)-	2.07e-1(2.64e-2)-	4.20e-1(3.51e-2)-	2.26e-1(1.79e-2)-	5.90e-1(1.61e-2)-	7.01e-1(5.72e-2)
IMF10	30	6.13e+1(1.76e+1)-	6.99e+1(1.20e+1)-	7.06e+1(8.84e+0)-	3.05e+1(9.21e+0)+	1.09e+2(1.89e+1)-	5.07e+1(1.04e+1)-	3.94e+1(4.22e+0)		30	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
	50	1.06e+2(2.03e+1)-	1.18e+2(2.36e+1)-	1.46e+2(2.62e+1)-	5.26e+1(1.18e+1)+	2.19e+2(2.22e+1)-	9.44e+1(1.51e+1)-	6.25e+1(4.97e+0)	IMF10	50	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
	100	3.03e+2(3.21e+1)-	3.23e+2(3.90e+1)-	4.12e+2(4.89e+1)-	1.33e+2(3.49e+1)≈	5.11e+2(5.13e+1)-	2.89e+2(5.16e+1)-	1.23e+2(2.48e+1)		100	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
	200	6.54e+2(8.88e+1)-	7.19e+2(5.55e+1)-	9.48e+2(5.80e+1)-	3.29e+2(8.25e+1)≈	1.18e+3(9.33e+1)-	7.29e+2(9.98e+1)-	4.00e+2(2.29e+2)		200	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
+/ - /	≈	8/31/1	0/39/1	7/31/2	14/20/6	0/40/0	8/32/0		+/-,	/≈	2/32/6	0/36/4	1/31/8	10/24/6	0/36/4	1/33/6	

#### General performance







#### Ablation study



GMOEA\* (the reproduction with pure genetic operators), GMOEA- (the reproduction with pure GAN operator), and GMOEA (the reproduction with the hybrid strategy).



The IGD Value achieved by GMOEA\*, GMOEA-, and GMOEA

- The pure GAN operator and the hybrid one perform significantly better than pure genetic operators on almost all the test instances.
- ➢ GMOEA outperforms GMOEA− on most test instances.

- Background
- Motivation
- Proposed Algorithm
- Experimental Results
- Conclusion and Future Work



#### **Conclusion**

- ➢ GANs work for EA
- > The hybrid of GANs and genetic operator is important
- **G** Future Work
  - Classification can be further improved
  - Diversity maintenance should be enhanced
  - Mode collapse in GANs should be addressed
  - More effective and efficient GANs can be used
  - > Applications of GMOEA in solving image processing tasks or optimization problems



# Thanks! (Q & A)